

# A Martingale Approach for Fractional Brownian Motions and Related Path Dependent PDEs

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## **Abstract:**

In this paper we study dynamic backward problems, with the computation of conditional expectations as a special objective, in a framework where the (forward) state process satisfies a Volterra type SDE, with fractional Brownian motion as a typical example. Such processes are neither Markov processes nor semimartingales, and most notably, they feature a certain time inconsistency which makes any direct application of Markovian ideas, such as flow properties, impossible without passing to a path-dependent framework. Our main result is a functional Ito formula, extending the seminal work of Dupire to our more general framework. In particular, unlike in his approach, where one needs only to consider the stopped paths, here we need to concatenate the observed path up to the current time with a certain smooth observable curve derived from the distribution of the future paths. This new feature is due to the time inconsistency involved in this paper. We then derive the path dependent PDEs for the backward problems. Finally, an application to option pricing and hedging in a financial market with rough volatility is presented. This is joint work with Jianfeng Zhang from the University of Southern California.