Singularity of random symmetric matrices

Letícia Mattos

IMPA

Abstract:
Let M_n be a uniformly-chosen random symmetric $n \times n$ matrix with entries in $\{-1, 1\}$. What is the probability for $\det(M_n) = 0$? A well-known conjecture states that the probability of this event is asymptotically equal to the probability that two of the rows or columns of M_n are equal (up to a factor of ± 1), and hence is equal to $\Theta(n^2 2^{-n})$. We develop an inverse Littlewood–Offord theorem in Z_p^n that applies under very mild conditions and made progress towards this conjecture, showing that the probability is bounded by $\exp(-c\sqrt{n})$.
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