Infinite-noise criticality

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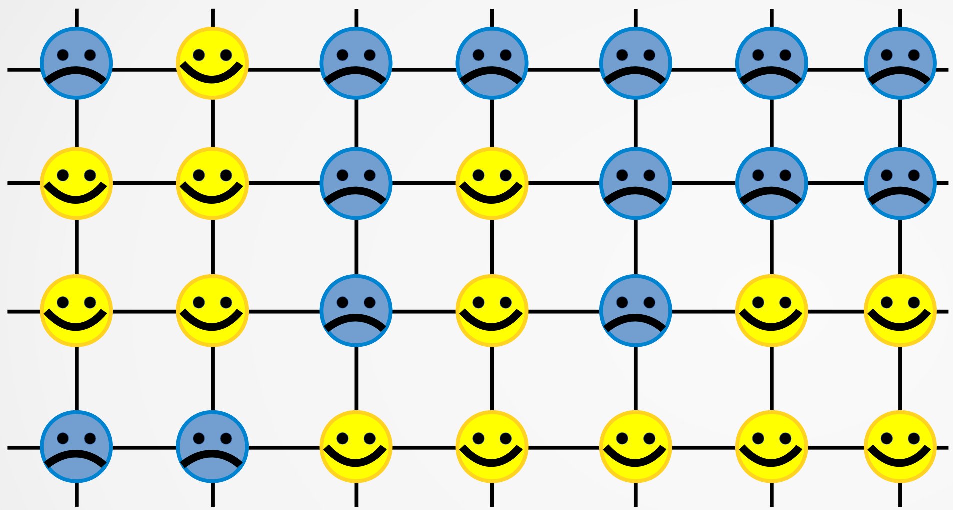
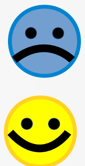
Outline

- Introduction: the contact process
  - Definition and applications to nonequilibrium PT
- Criticality on a fluctuating (noisy) environment
  - Infinite-noise criticality at $d=\infty$: random walk
  - Infinite-noise criticality at finite d: real-time RG
- Observables and critical behaviour
- Conclusions

The contact process: definition

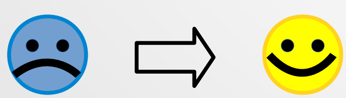
Prototypical model for epidemic spreading
- T. E. Harris, Ann. Prob. **2**, 969 (1974)



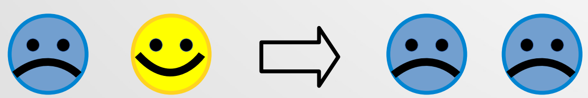


ACTIVE (infected)
INACTIVE (healthy)

Stochastic dynamics:



Decay (healing) rate: μ



Offspring production (infection) rate: $\lambda \times \frac{\text{no. sick neighbors}}{\text{coordination}}$

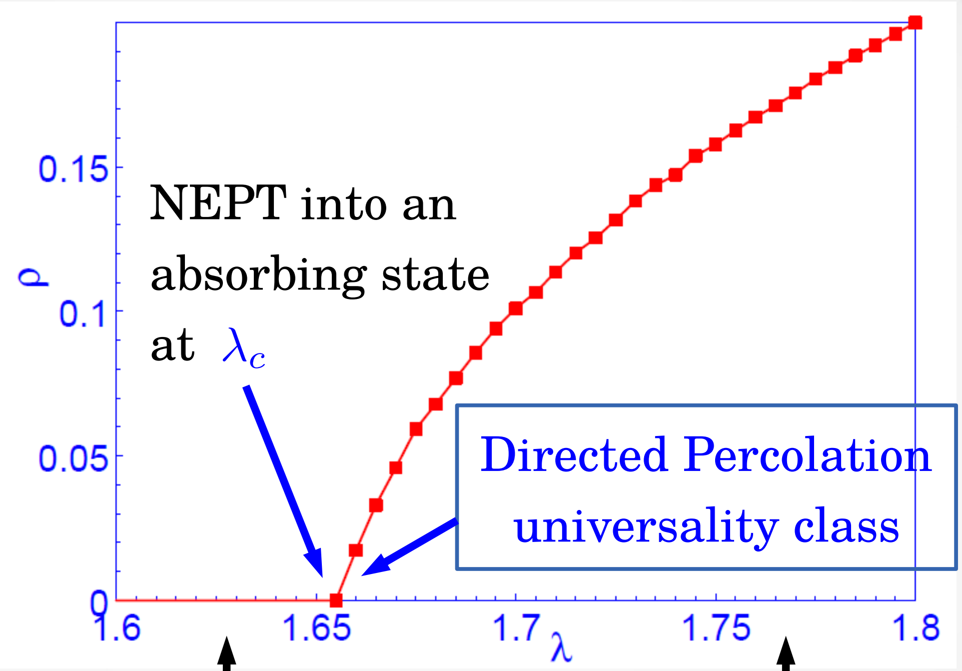
Active sites steady-state density: $\rho \triangleq \lim_{t,V\to\infty} \mathbb{E}(\rho(V,t))$

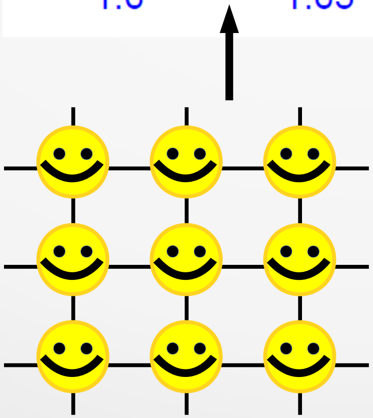
The contact process: noneq. PT

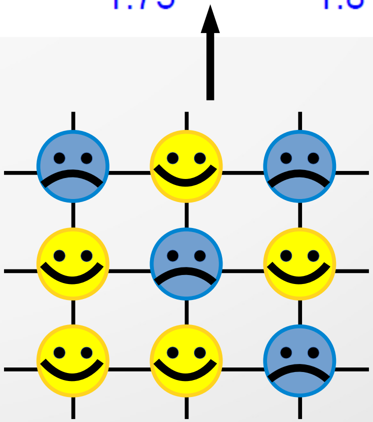
Active sites

steady-state density:

$\rho = \lim_{t,V\to\infty} \rho(V,t)$



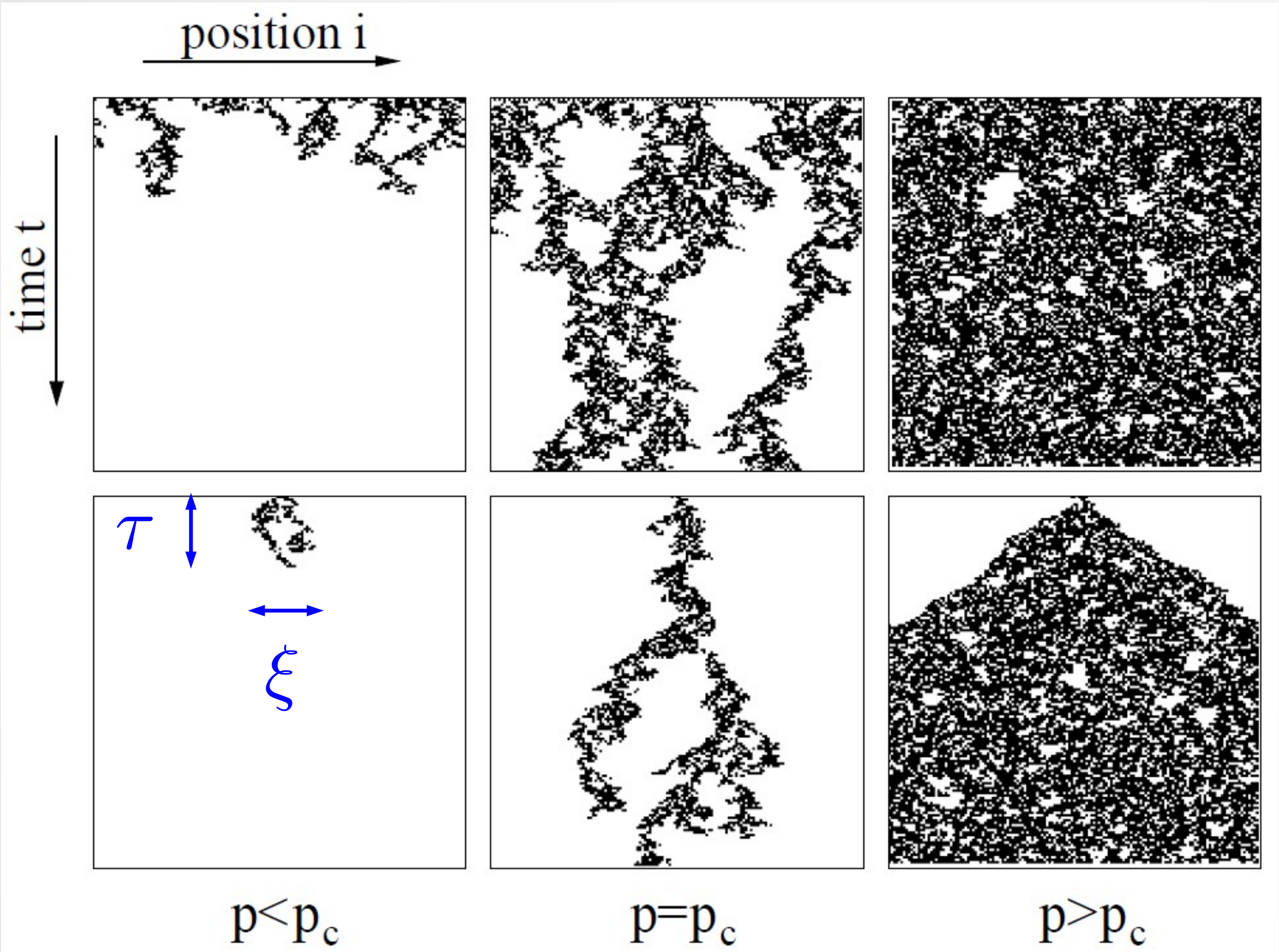




Absorbing (inactive) state

Active (fluctuating) steady state

The contact process: DP universality class



2nd order PT

$$\rho \sim (\lambda - \lambda_c)^{\beta}$$

$$\xi \sim |\lambda - \lambda_c|^{-\nu_\perp}$$

$$\tau \sim |\lambda - \lambda_c|^{-\nu_\parallel}$$

$$\tau \sim \xi^{z}$$

$$\rho \sim t^{-\delta}$$

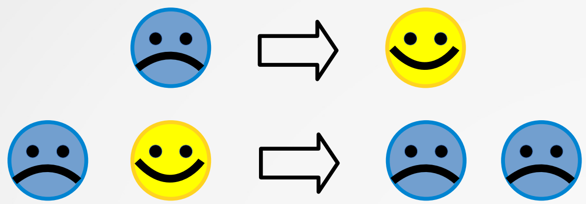
Percolation and directed percolation: geometric PTs

The CP can be mapped on a $d+1$ DP geometric problem

Picture credit:
Hinrichsen, Adv. Phys. **49,** 815 (2000)

Our model

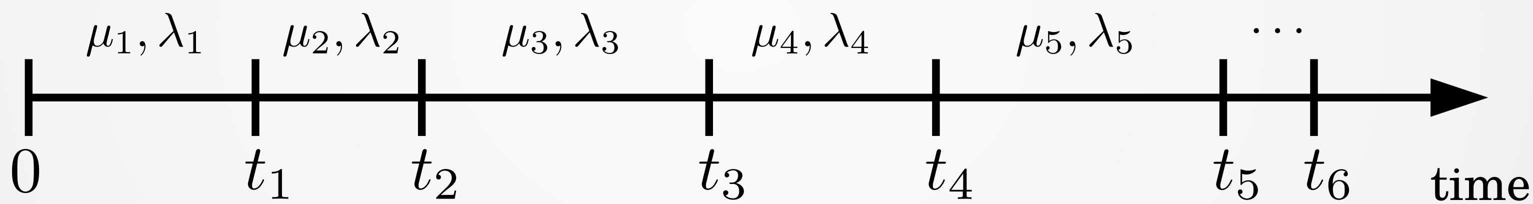
Contact process in a fluctuating (noisy) environment:



Healing rate: μ

Infection rate: λ

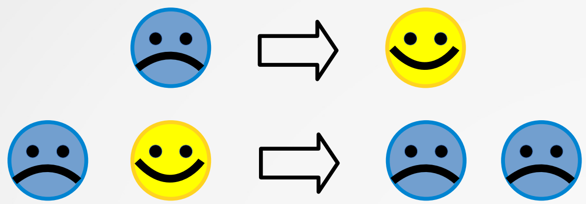
Time-dependent iid



Time intervals $\Delta t_i = t_i - t_{i-1}$ are finite.

What do we want to know?

Contact process in a fluctuating (noisy) environment:



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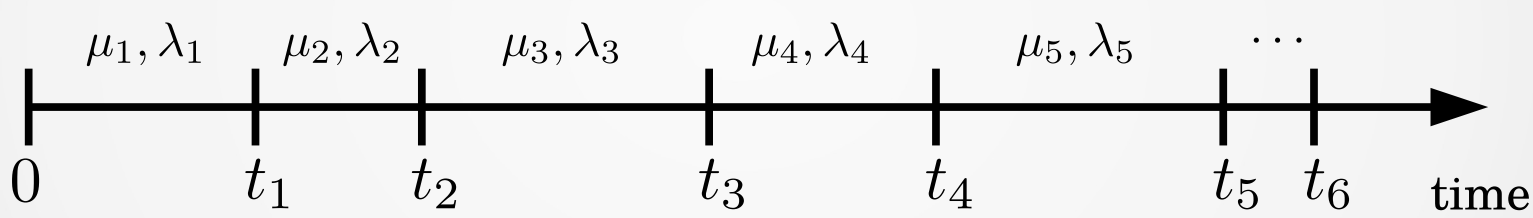


Density PDF as a function of time?

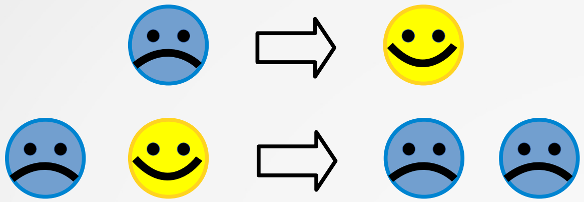
Is the mean density representative of the many possible time histories?

Lifetime of finite systems?

Correlation length/time?

Our model: limiting cases

Contact process in a fluctuating (noisy) environment:



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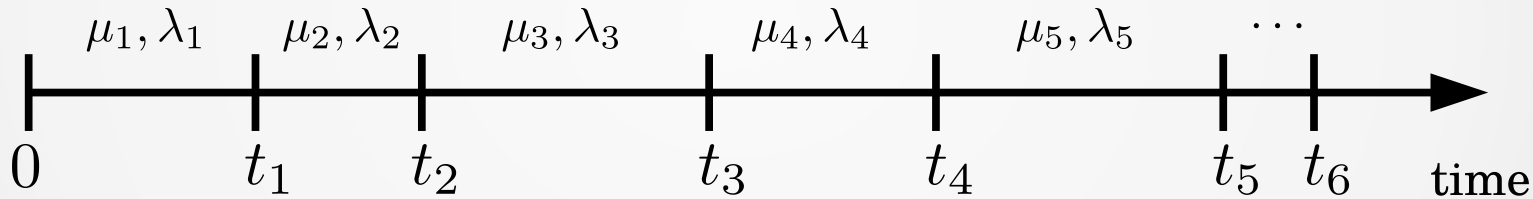


In all time intervals, the system is in the ACTIVE phase: $\lambda_i > \lambda_{c,i} \sim \mu_i$

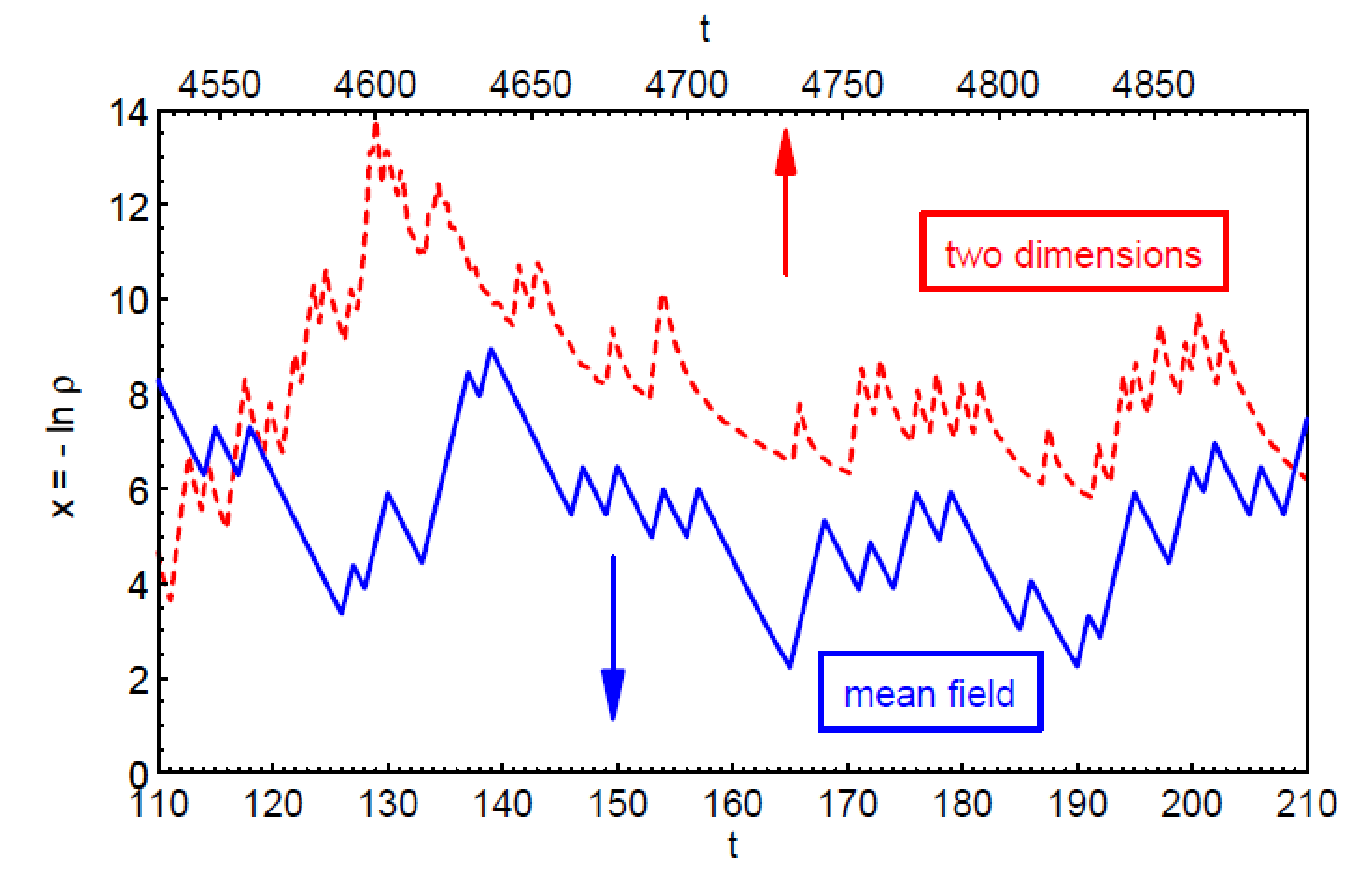
ABSORBING phase: $\lambda_i < \lambda_{c,i}$

Then, usual dynamics. Time disorder has "little" effects.

Interesting case: "time patches" are in opposite phases – phase transition.

What do we expect?

Large density fluctuations – field-theoretical methods are not appropriate.



Monte Carlo time evolution of the density at criticality.

Mean field approach

How do we face the problem? Let's try something simple.

Langevin Eq. in MF level: $\partial_t \rho = -\mu_i \rho + \lambda_i \rho(1-\rho) = (\lambda_i - \mu_i)\rho - \lambda_i \rho^2, \quad t_{i-1} < t < t_i$

Nonlinear term preventing ρ>1

Integrating: $\rho_{i+1}^{-1} = A_i \rho_i^{-1} + B_i,$

$$A_i = e^{(\mu_i - \lambda_i)\Delta t_i}$$
$$B_i = \frac{\lambda_i}{\mu_i - \lambda_i}(A_i - 1) \longrightarrow$$

Lets neglect it and recover linearity

Let $\quad x_i = -\ln \rho_i, \qquad x_{i+1} = x_i + \ln A_i \qquad$ RANDOM WALK

What is the role played by B_i? - To ensure the walker is in the positive side $x_i \geq 0$

How do we put the nonlinearity back in the game?

RANDOM WALK with a reflecting hard wall

Mean field approach: Phase transition

$$x = -\ln\rho$$
$$x_{i+1} = x_i + \ln A_i = x_i + (\mu_i - \lambda_i)\Delta t_i$$

$P(x,t) \equiv$ Prob. of finding the walker between $[x, x+dx]$ at time $t = \sum_i \Delta t_i$

RANDOM WALK with a reflecting hard wall

Solve $\partial_t P(x,t) = \frac{1}{2}\sigma^2\partial_x^2 P - v\partial_x P$

Hard wall constraint $vP - \frac{1}{2}\sigma^2\partial_x P = 0$ at $x=0$.

Mean field approach: Phase transition

$$x = -\ln \rho$$
$$x_{i+1} = x_i + \ln A_i = x_i + (\mu_i - \lambda_i)\Delta t_i$$

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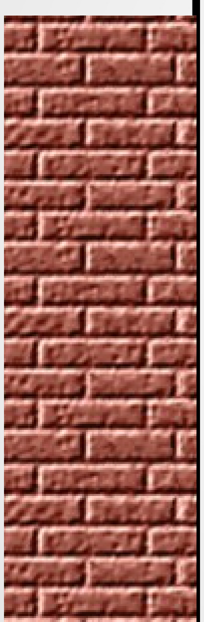


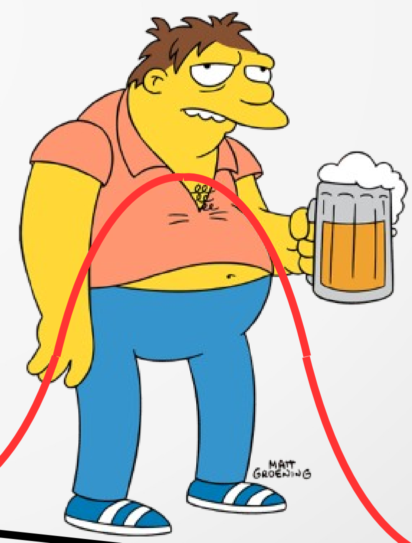
Usual Gaussian

$$\langle x \rangle = vt$$
$$\sigma_x = \sigma\sqrt{t}$$

Implications:

$$\langle \rho(t) \rangle \sim e^{-t/\tau}$$

$$\tau \sim v^{-\nu_\parallel}, \Rightarrow \boxed{\nu_\parallel = 2}$$



0

x

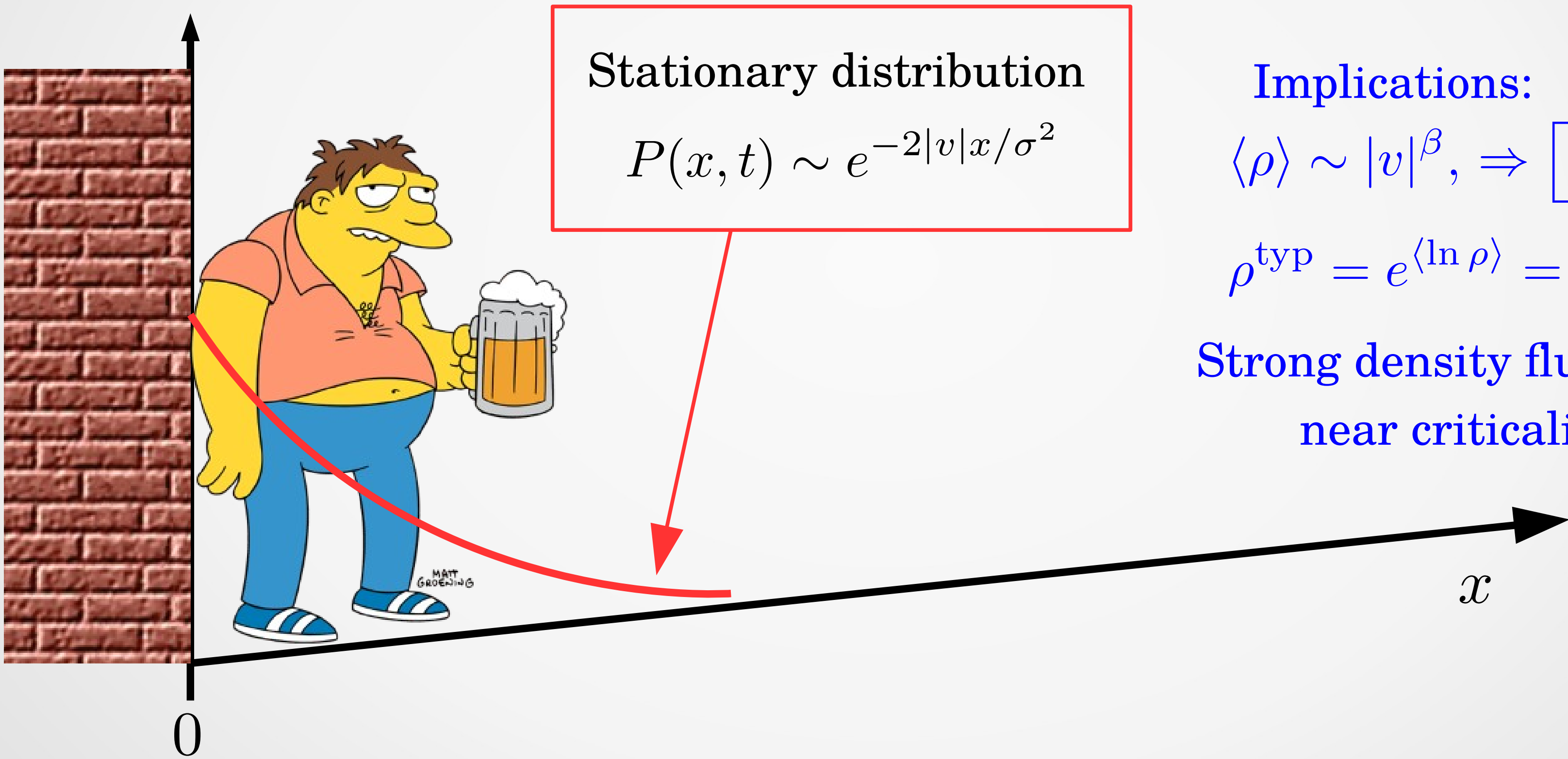
RANDOM WALK with a
reflecting hard wall

1st regime: Inactive phase $v \equiv \langle \mu_i - \lambda_i \rangle > 0$

Mean field approach: Phase transition

$$x = -\ln\rho$$
$$x_{i+1} = x_i + \ln A_i = x_i + (\mu_i - \lambda_i)\Delta t_i$$

$P(x,t) \equiv$ Prob. of finding the walker between [x,$x+dx$] at time $t = \sum_i \Delta t_i$



Implications:

$\langle\rho\rangle \sim |v|^\beta, \Rightarrow \boxed{\beta = 1}$

$\rho^{\text{typ}} = e^{\langle\ln\rho\rangle} = e^{-\sigma^2/(2|v|)}$

Strong density fluctuations near criticality $v\to 0$

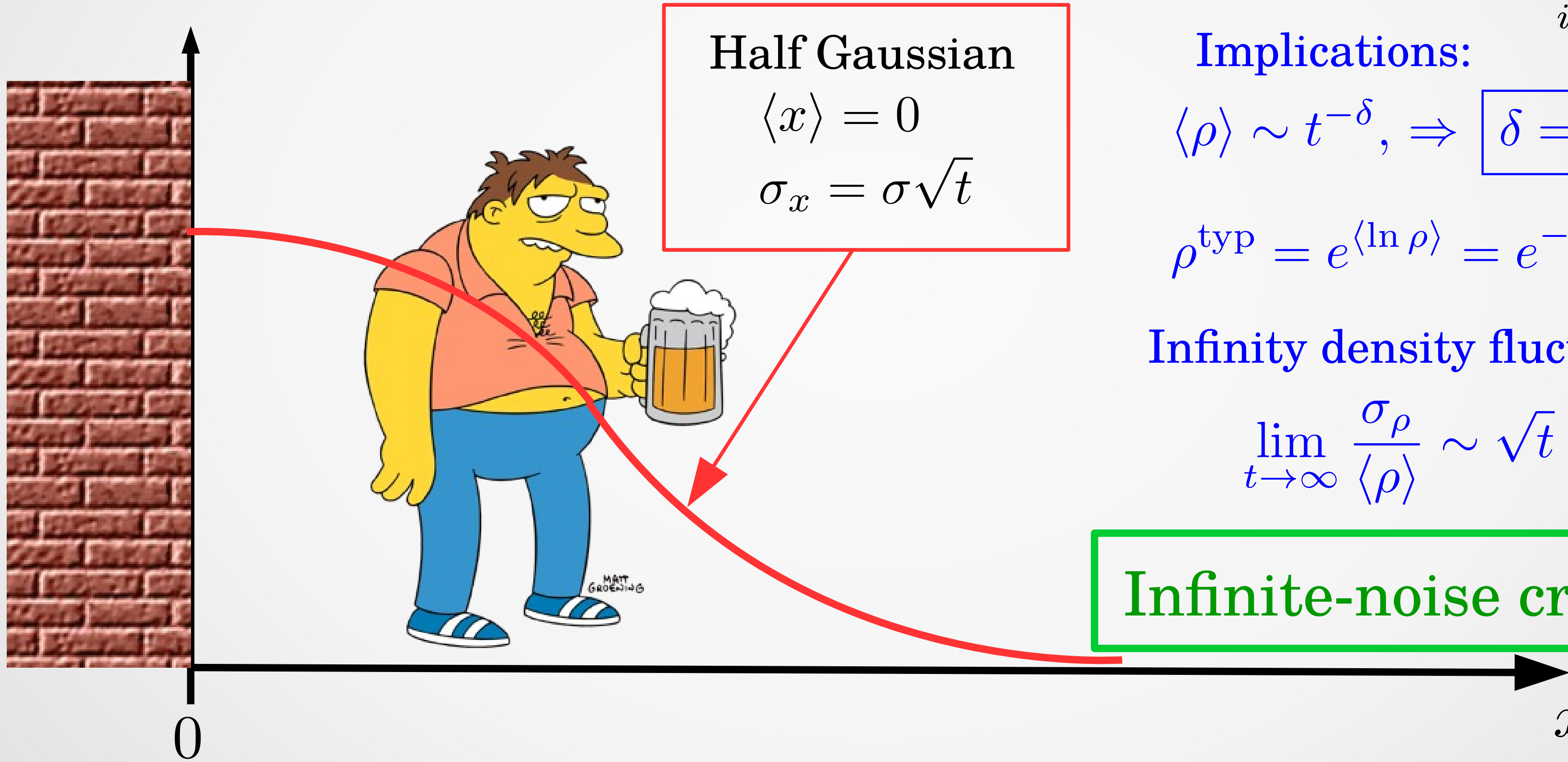
RANDOM WALK with a reflecting hard wall

2nd regime: Active phase $\quad v \equiv \langle\mu_i - \lambda_i\rangle < 0$

Mean field approach: Phase transition

$$x = -\ln \rho$$
$$x_{i+1} = x_i + \ln A_i = x_i + (\mu_i - \lambda_i)\Delta t_i$$

$P(x,t) \equiv$ Prob. of finding the walker between $[x, x+dx]$ at time $t = \sum_i \Delta t_i$



Implications:

$\langle \rho \rangle \sim t^{-\delta}, \Rightarrow \boxed{\delta = 1/2}$

$$\rho^{\text{typ}} = e^{\langle \ln \rho \rangle} = e^{-\sqrt{\frac{2\sigma^2}{\pi} t}}$$

Infinity density fluctuations

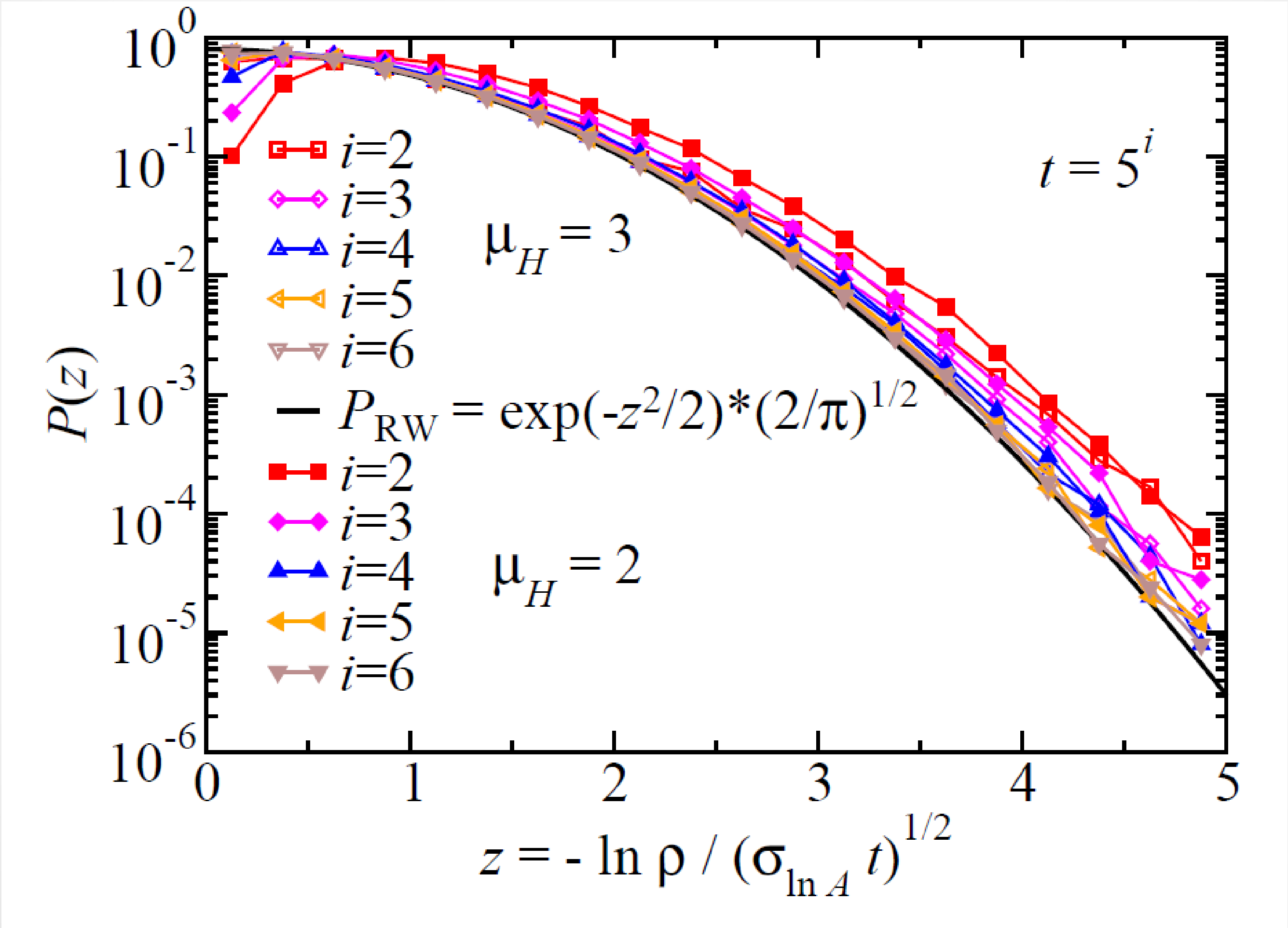
$$\lim_{t\to\infty} \frac{\sigma_\rho}{\langle \rho \rangle} \sim \sqrt{t} = \infty$$

Infinite-noise criticality

RANDOM WALK with a reflecting hard wall

3rd regime: Critical point $v \equiv \langle \mu_i - \lambda_i \rangle = 0$

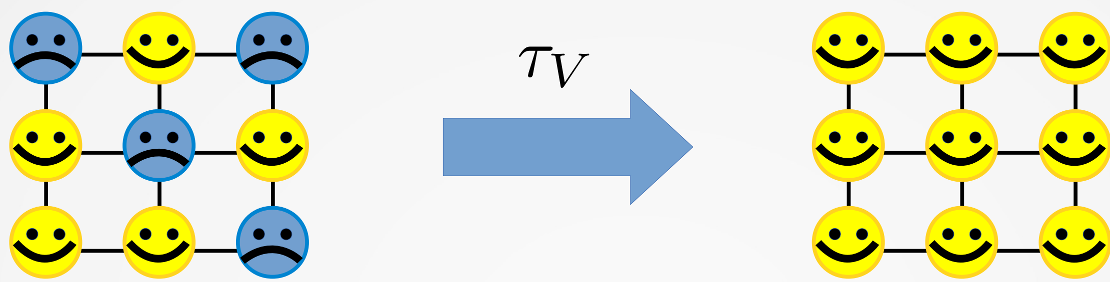
Mean field: comparison with numerics

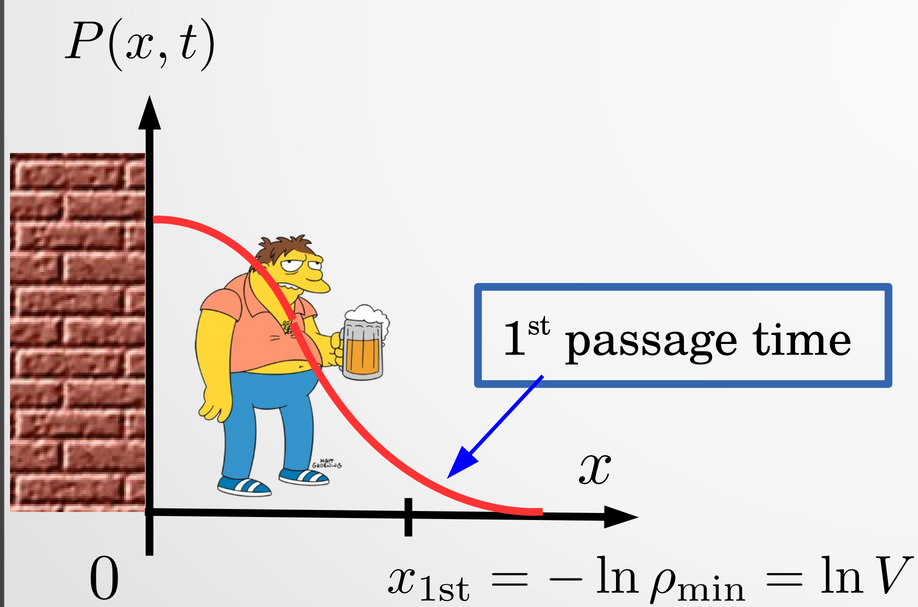


Half Gaussian at criticality: in agreement with our expectations

Mean field approach: Lifetime

Any finite system eventually decays into the absorbing state





Active phase

$$\tau_V^{-1} \sim \int_{x_{1\mathrm{st}}}^{\infty} \mathrm{d}x P(x) = \int_0^{1/V} \mathrm{d}\rho P_\rho(\rho) \sim V^{-2|v|/\sigma^2}$$

Criticality

$$\rho^{\mathrm{typ}}(\tau_V) = 1/V, \Rightarrow \tau_V \sim \ln^2 V$$

Inactive phase

$$\rho^{\mathrm{typ}}(\tau_V) = 1/V, \Rightarrow \tau_V \sim \ln V$$

Mean field approach: What are we doing?

From complicated Langevin

$$\partial_t \rho(\mathbf{r},t) = (\lambda - \mu)\rho - \lambda\rho^2 + D\nabla^2\rho + \zeta(\mathbf{r},t)$$

to simple logistic equation

$$\partial_t \rho = (\lambda_i - \mu_i)\rho - \lambda_i \rho^2,$$

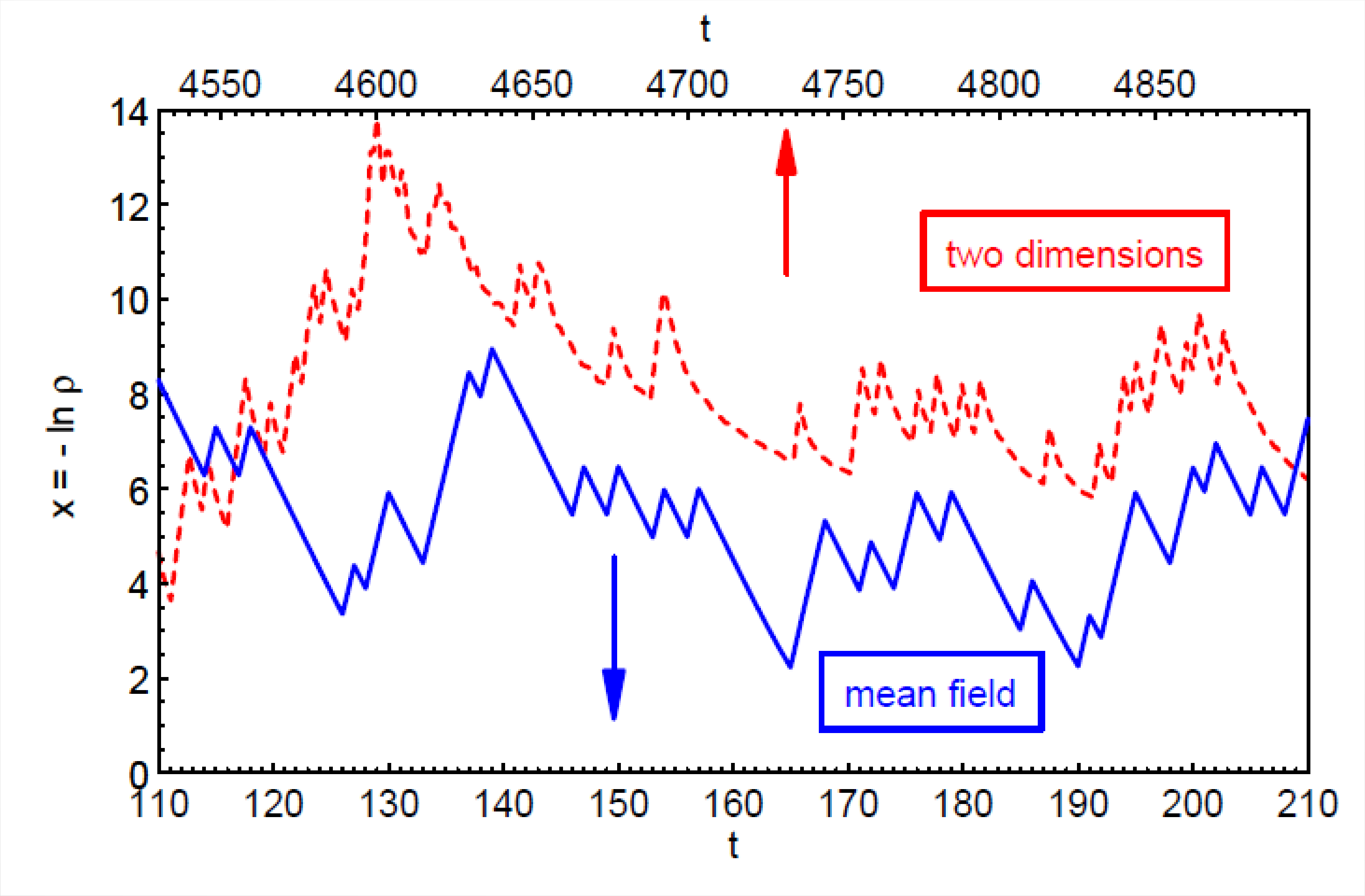
1. This logistic equation is already interesting on its own. It models processes in which diffusive mixing is strong enough to wipe out spatial structures
   - Biological population, chemical reactions, linguistics, etc...

2. For the CP at finite dimensions, spatial correlations are important.
   - How do we account for them? - Real-time RG

Does the infinite-noise criticality survive at finite d?

Real-time RG

Idea: coarse-grain small time structures



Real-time RG

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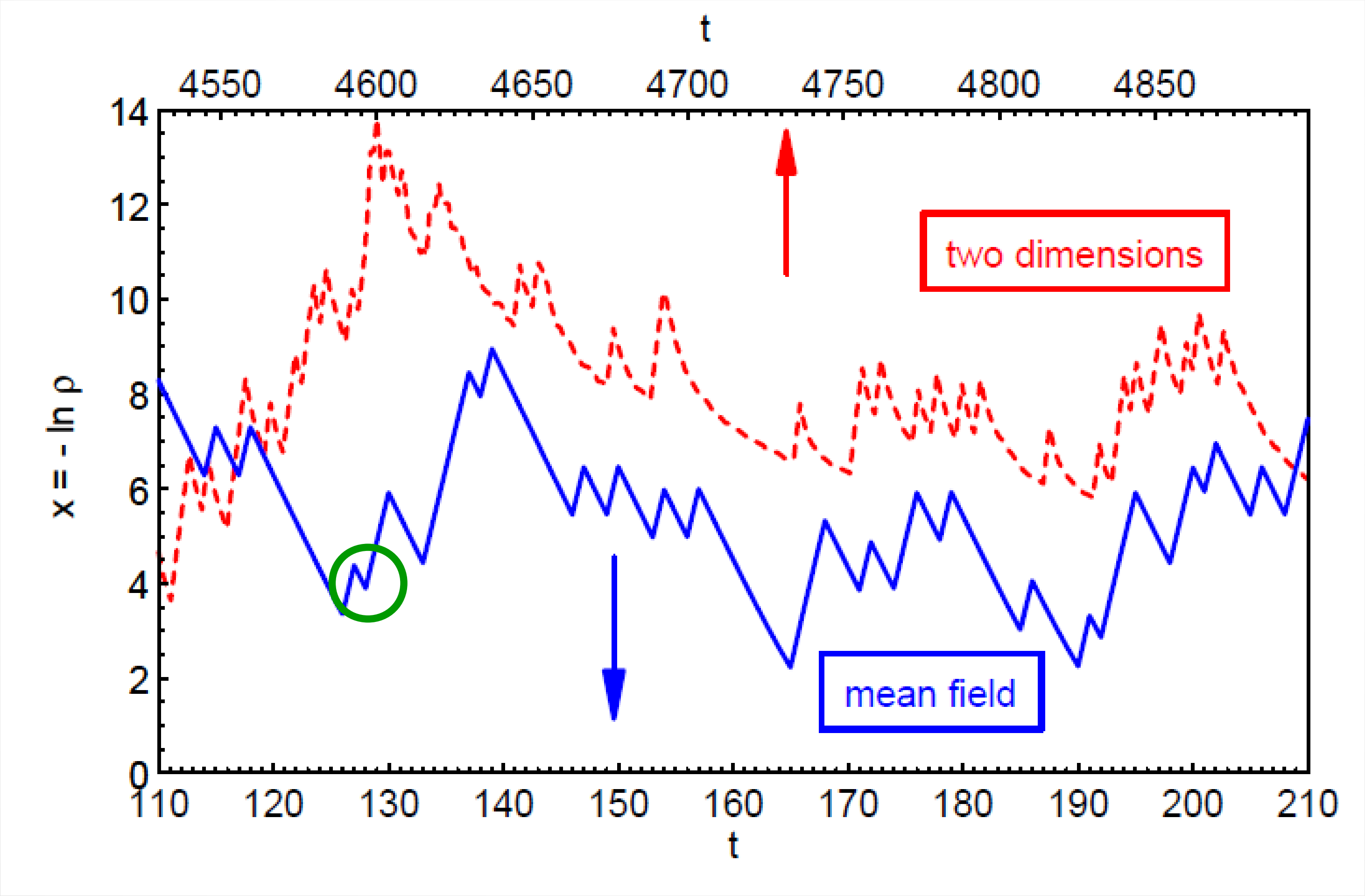

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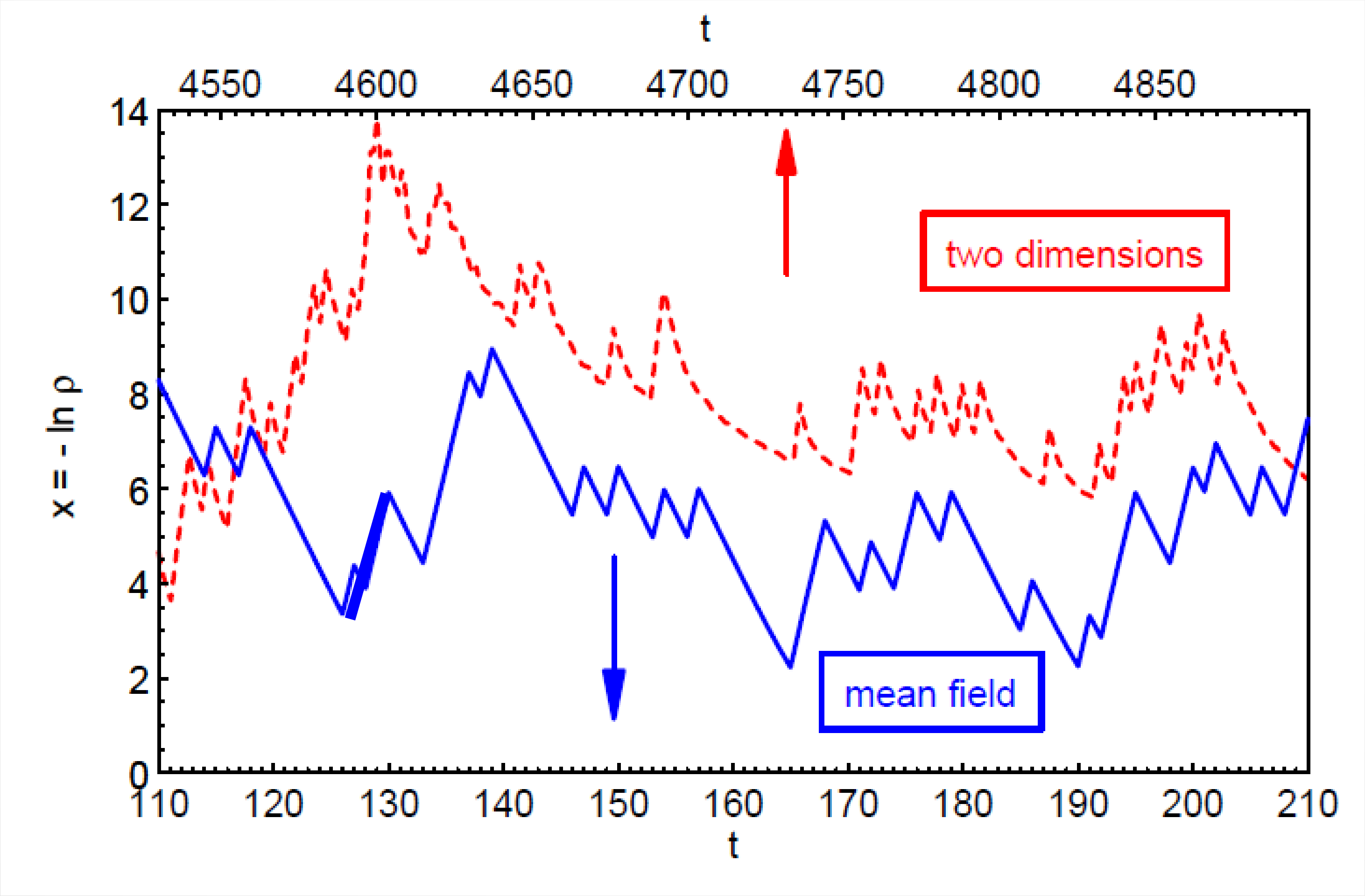

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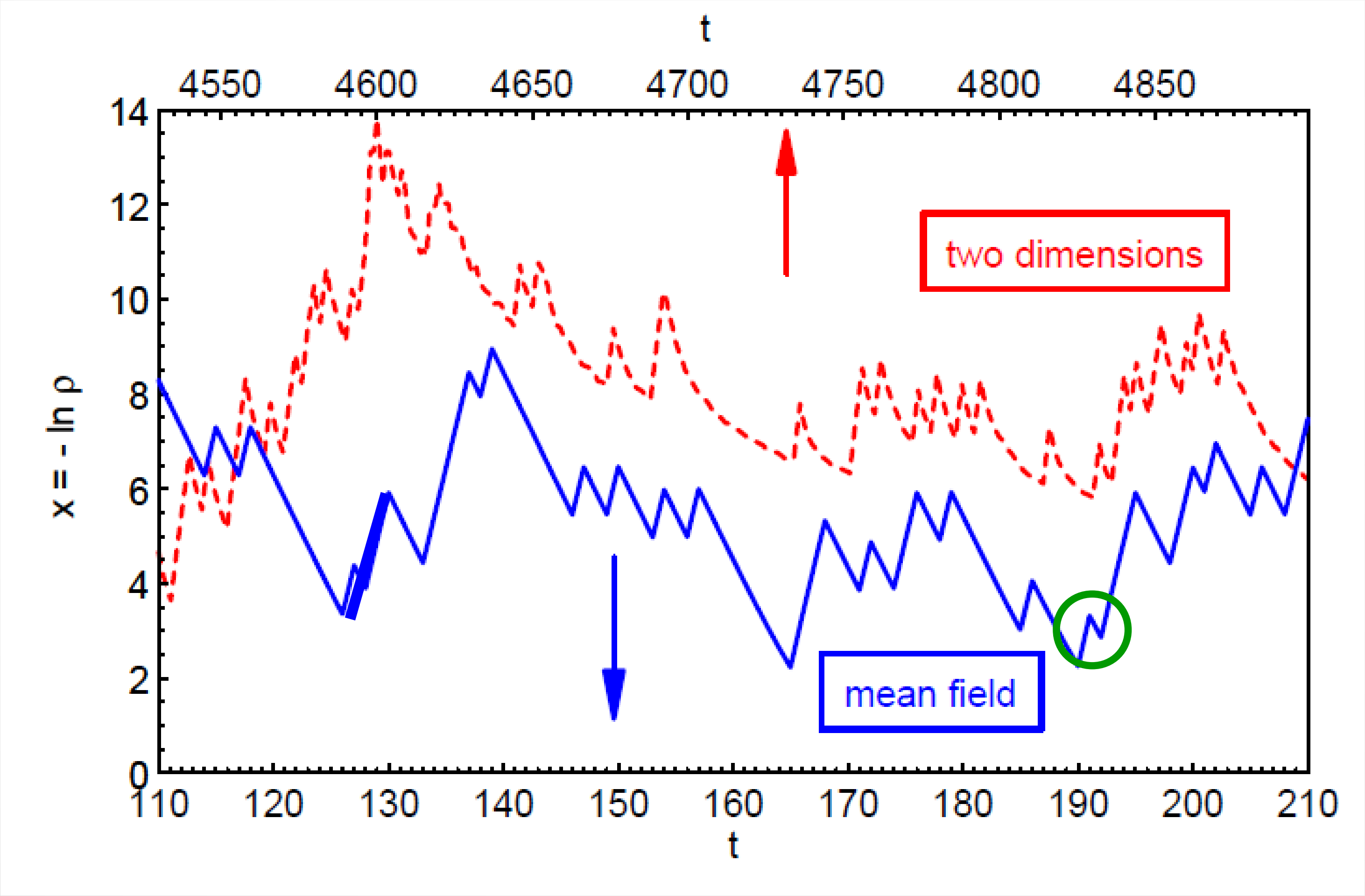

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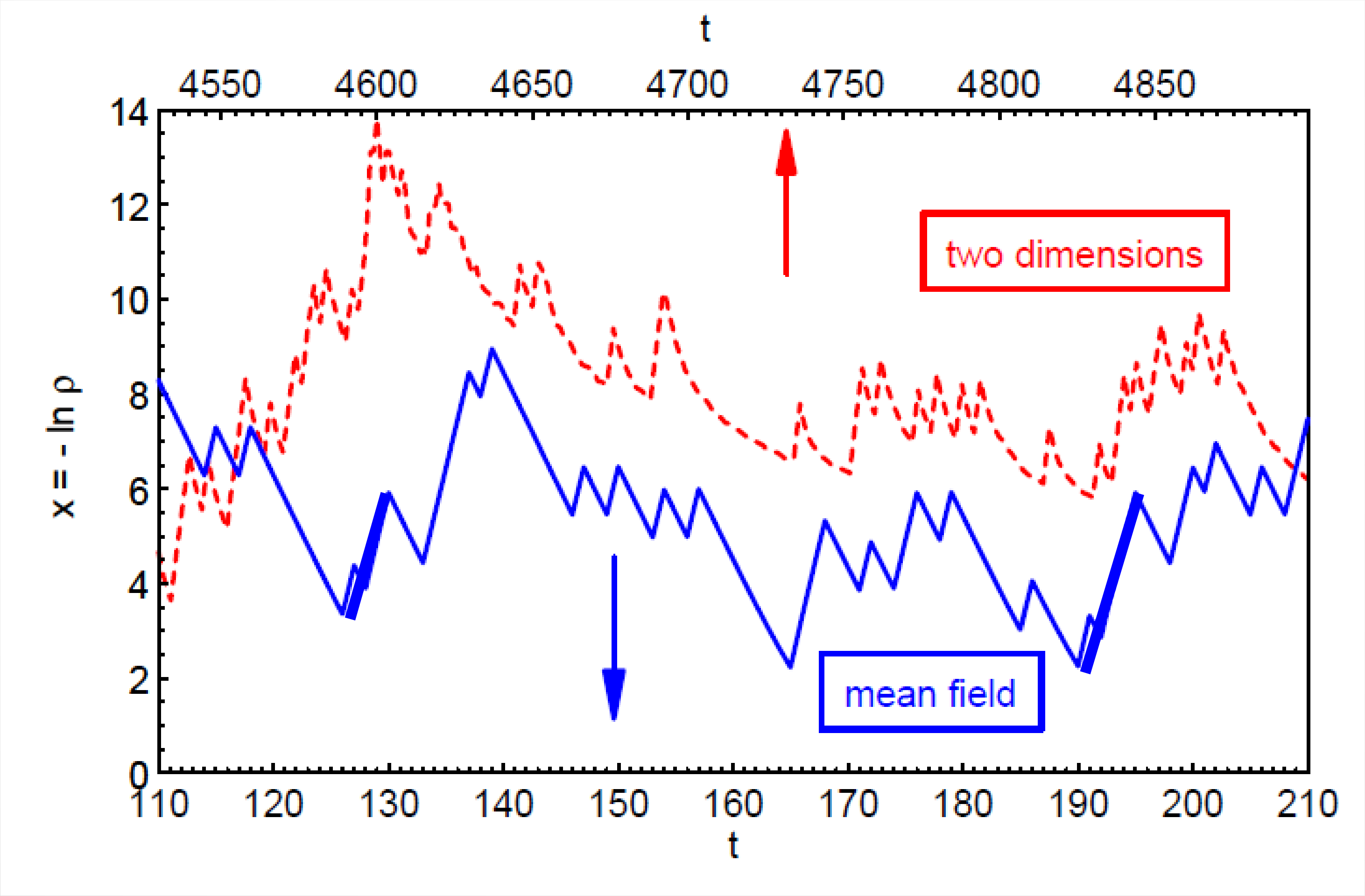

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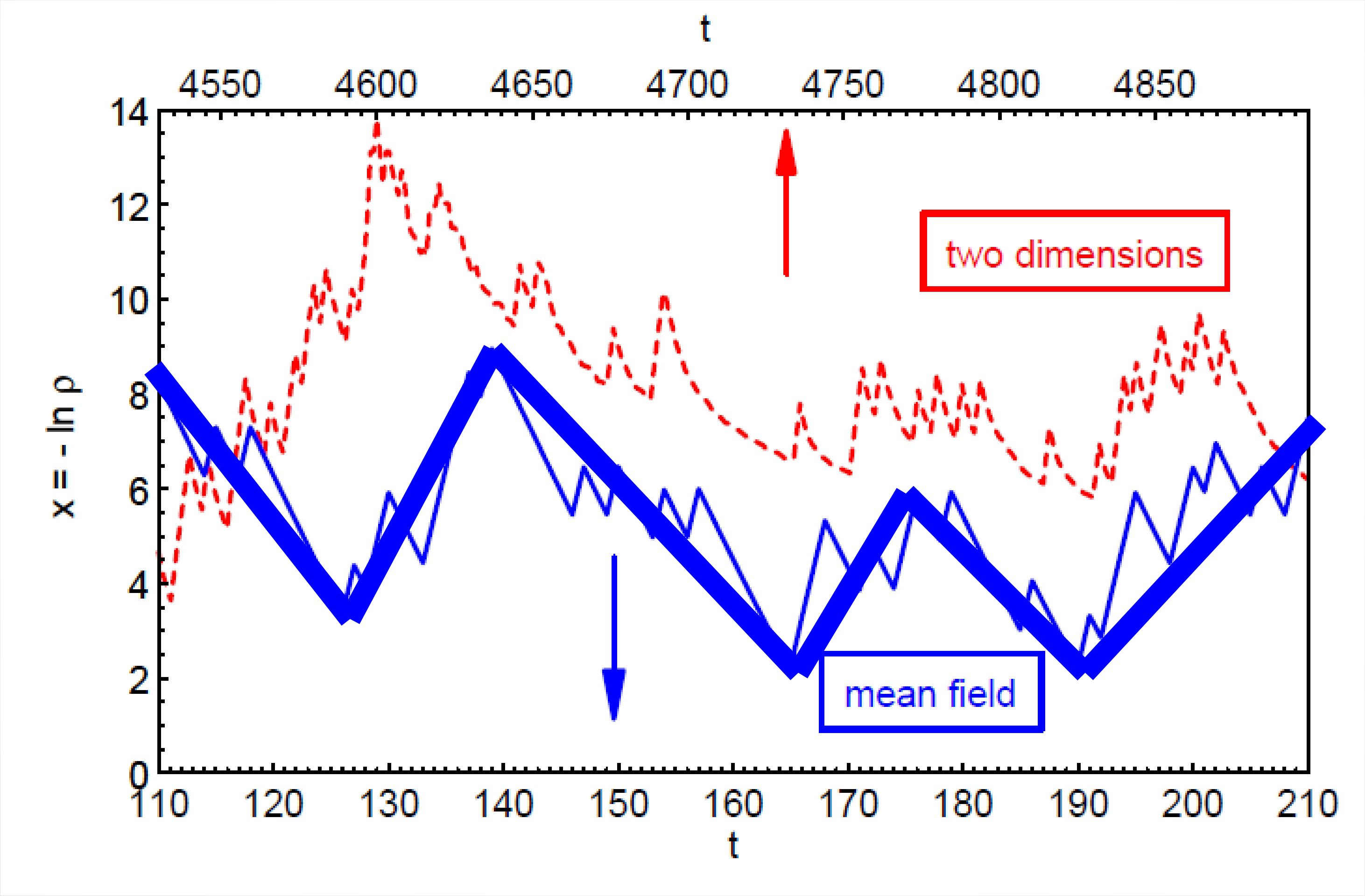

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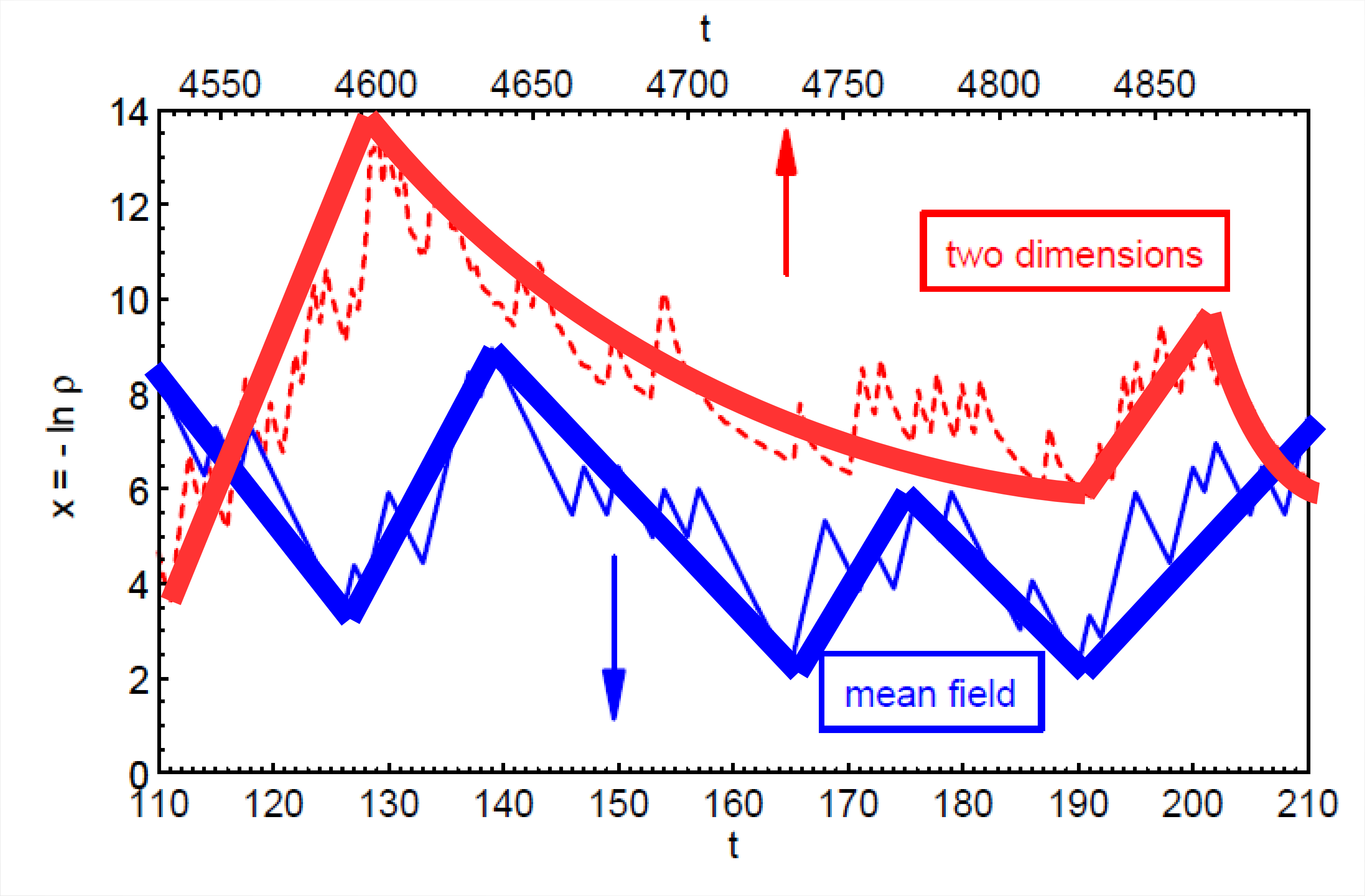

Real-time RG

Idea: coarse-grain small time structures

How do we do it? Eliminate the smallest upward and downward segments

Iterative map: $\rho_{i+1}^{-1} = A_i \rho_i^{-1} + B_i,$ not much important for small densities

Upward segment: $A_i = A_i^{\mathrm{up}} > 1$

Downward segment: $A_i = A_i^{\mathrm{down}} < 1$

Cutoff: $\Omega = \min\{A_i^{\mathrm{up}}, 1/A_i^{\mathrm{down}}\}$

Real-time RG: decimation procedure

Idea: coarse-grain small time structures

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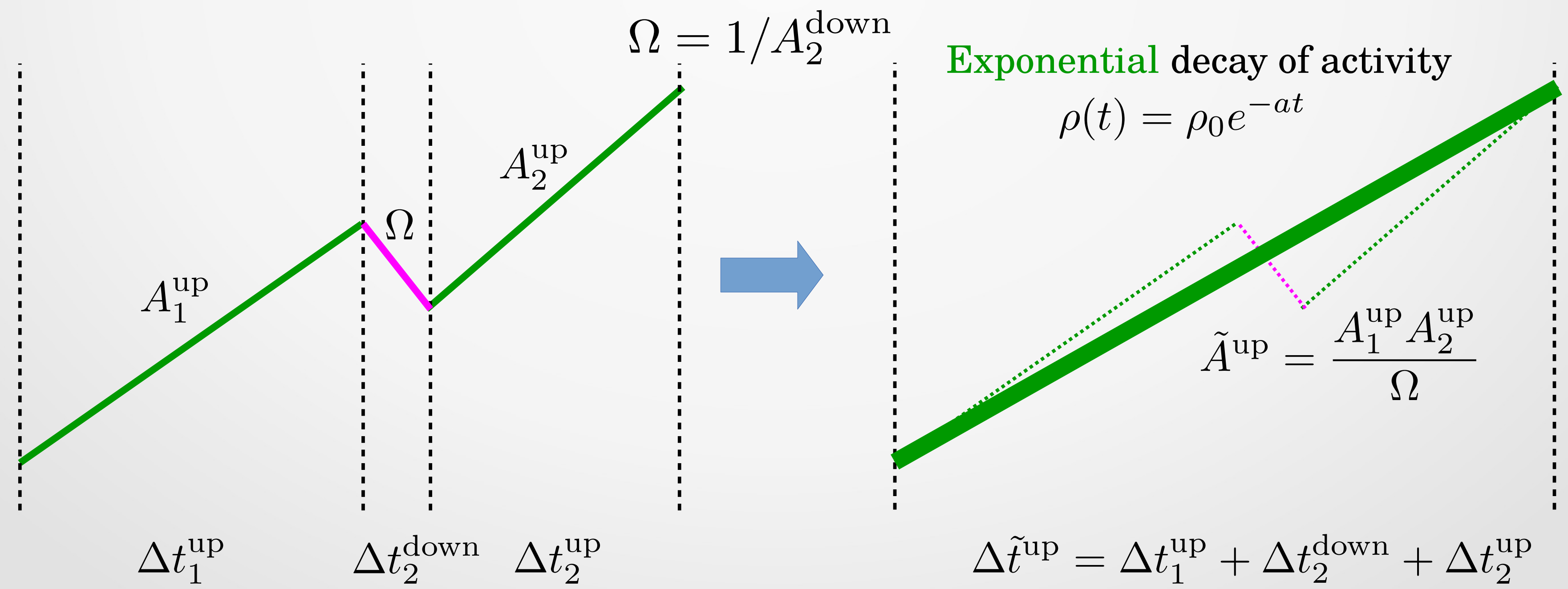

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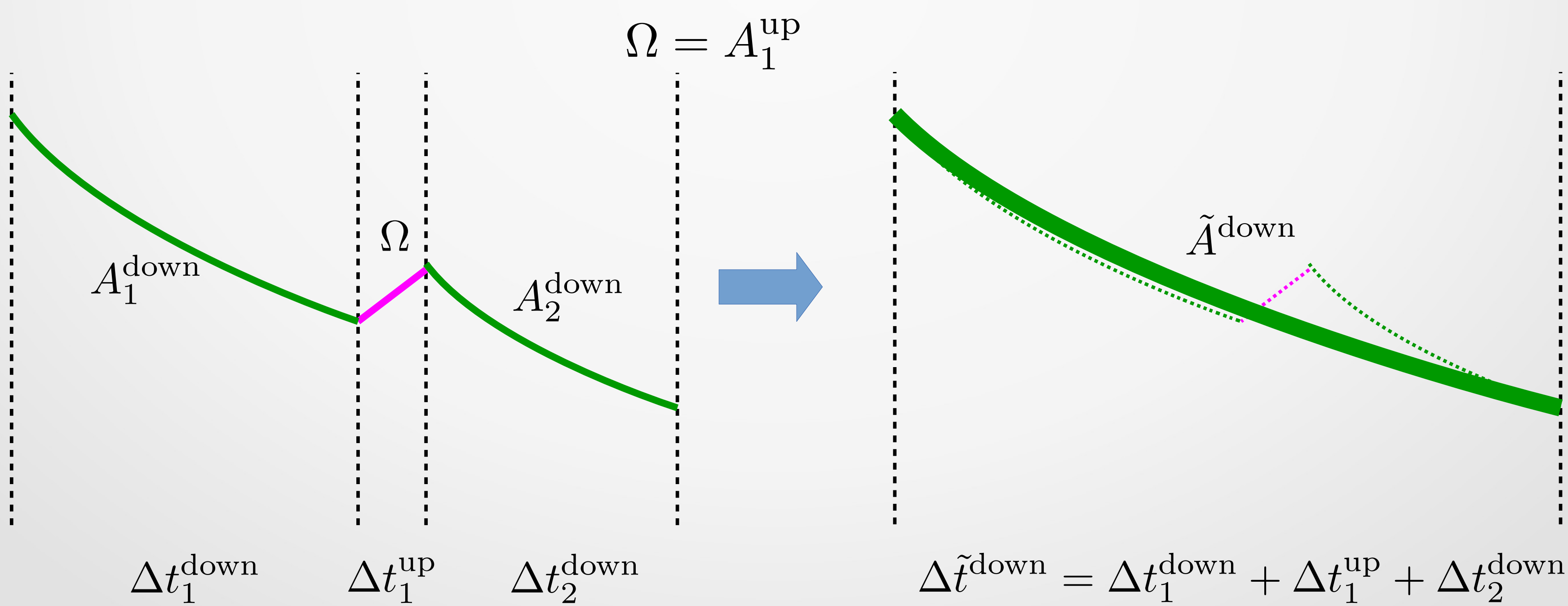

Real-time RG: decimation procedure

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Upward segment: $A_i = A_i^{\mathrm{up}} > 1$

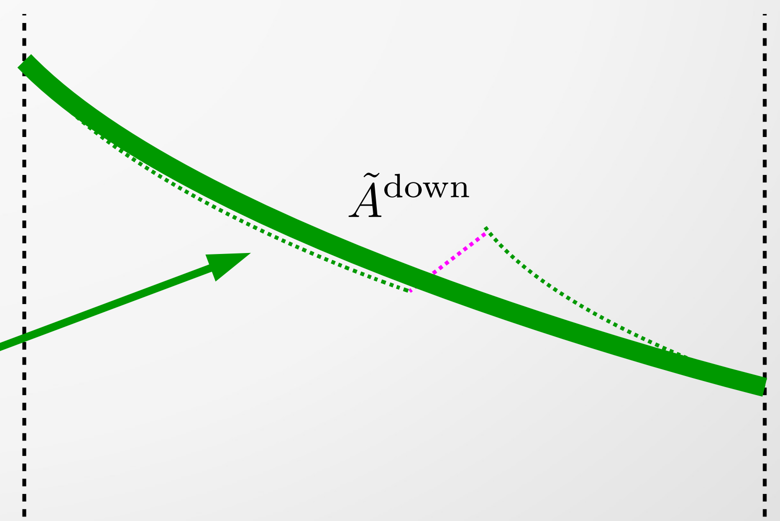
Downward segment: $A_i = A_i^{\mathrm{down}} < 1$

Cutoff: $\Omega = \min\{A_i^{\mathrm{up}}, 1/A_i^{\mathrm{down}}\}$

$$\Omega = A_1^{\mathrm{up}}$$

Mean field: exponential activity spreading

$$(\tilde{A}^{\mathrm{down}})^{-1} = \frac{(A_1^{\mathrm{down}})^{-1}(A_2^{\mathrm{down}})^{-1}}{\Omega}$$



Finite d:

$$(\tilde{A}^{\mathrm{down}})^{-\frac{1}{d}} = (A_1^{\mathrm{down}})^{-\frac{1}{d}} + (A_2^{\mathrm{down}})^{-\frac{1}{d}} - \Omega^{-\frac{1}{d}}$$

Maximum spreading: ballistic $\rho(t) = \rho_0(1+bt)^d$

Real-time RG: the RG flow

Idea: Follow the distributions of A^{up} and A^{down} as $\Omega \to \infty$

Variable changing:
$$\Gamma = \ln \Omega$$
$$\beta = \ln A^{\text{up}}\Omega$$
$$\zeta = d[(\Omega A^{\text{down}})^{-1/d} - 1]$$

Flow equations:

$$\partial_\Gamma \mathcal{R} = \partial_\beta \mathcal{R} + (\mathcal{R}_0 - \mathcal{P}_0)\mathcal{R} + \mathcal{P}_0 \mathcal{R} \overset{\beta}{\otimes} \mathcal{R}$$
$$\partial_\Gamma \mathcal{P} = \left(1 + \frac{\zeta}{d}\right)\partial_\zeta \mathcal{P} + \left(\mathcal{P}_0 - \mathcal{R}_0 + \frac{1}{d}\right)\mathcal{P} + \mathcal{R}_0 \mathcal{P} \overset{\zeta}{\otimes} \mathcal{P}$$

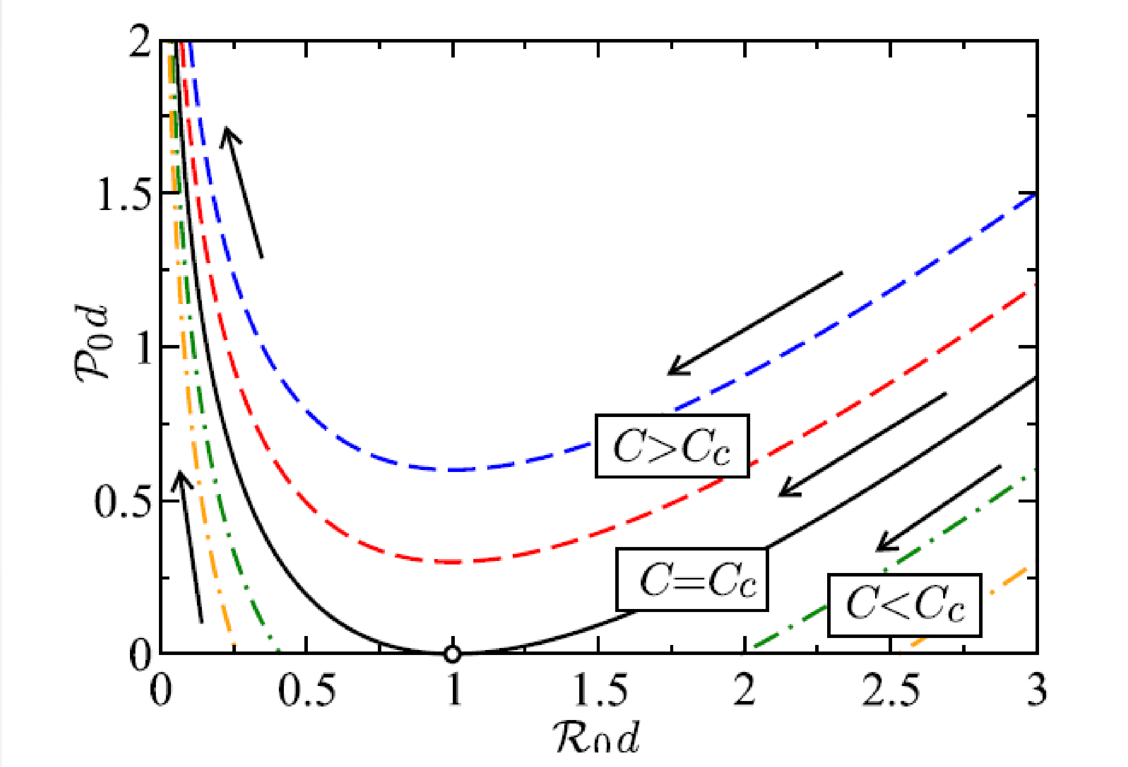
Fixed point solutions:

$$\mathcal{R}(\beta, \Gamma) = \mathcal{R}_0 e^{-\mathcal{R}_0 \beta}$$
$$\mathcal{P}(\zeta, \Gamma) = \mathcal{P}_0 e^{-\mathcal{P}_0 \zeta}$$

"Korsterlitz-Thouless" RG flow

$$\frac{\mathrm{d}}{\mathrm{d}\Gamma}\mathcal{R}_0 = -\mathcal{R}_0 \mathcal{P}_0$$
$$\frac{\mathrm{d}}{\mathrm{d}\Gamma}\mathcal{P}_0 = \left(\frac{1}{d} - \mathcal{R}_0\right)\mathcal{P}_0$$

Real-time RG: the RG flow



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$$\frac{\mathrm{d}}{\mathrm{d}\Gamma}\mathcal{P}_0 = \left(\frac{1}{d} - \mathcal{R}_0\right)\mathcal{P}_0$$

RG trajectories:

$$\mathcal{P}_0 = \mathcal{R}_0 - \frac{1}{d}\ln\mathcal{R}_0 + C$$

Real-time RG: the phases

INACTIVE phase

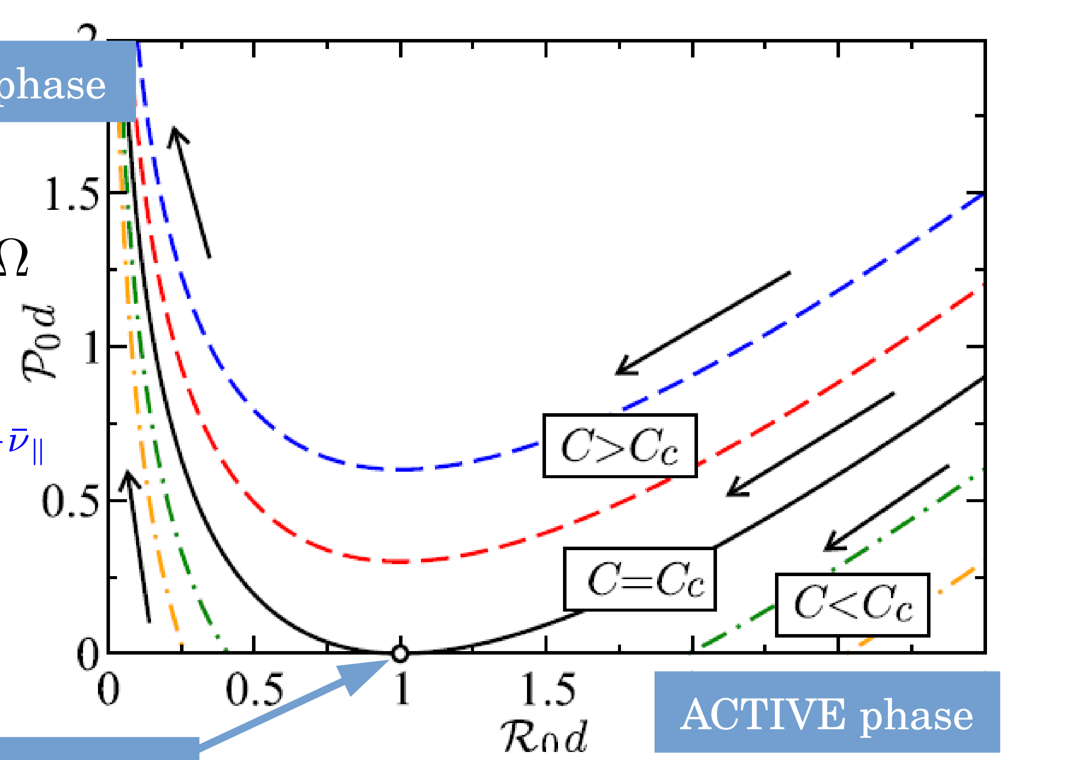
$\langle A^{\mathrm{up}} \rangle \to \infty$

$\left\langle 1/A^{\mathrm{down}} \right\rangle \to \Omega$

$\langle \rho(t) \rangle \sim e^{-t/\tau}$

$\ln \tau \sim (C - C_c)^{-\bar{\nu}_\parallel}$

$\boxed{\bar{\nu}_\parallel = 2}$



Criticality

ACTIVE phase

$\langle A^{\mathrm{up}} \rangle \to \Omega$

$\left\langle 1/A^{\mathrm{down}} \right\rangle \to \infty$

$\langle \rho(t) \rangle \sim (\ln t)^{-\bar{\delta}}, \Rightarrow \boxed{\bar{\delta} = 1}$

$\langle \rho \rangle \sim (C_c - C)^{\beta}, \Rightarrow \boxed{\beta = \frac{1}{2}}$

Infinite-noise criticality

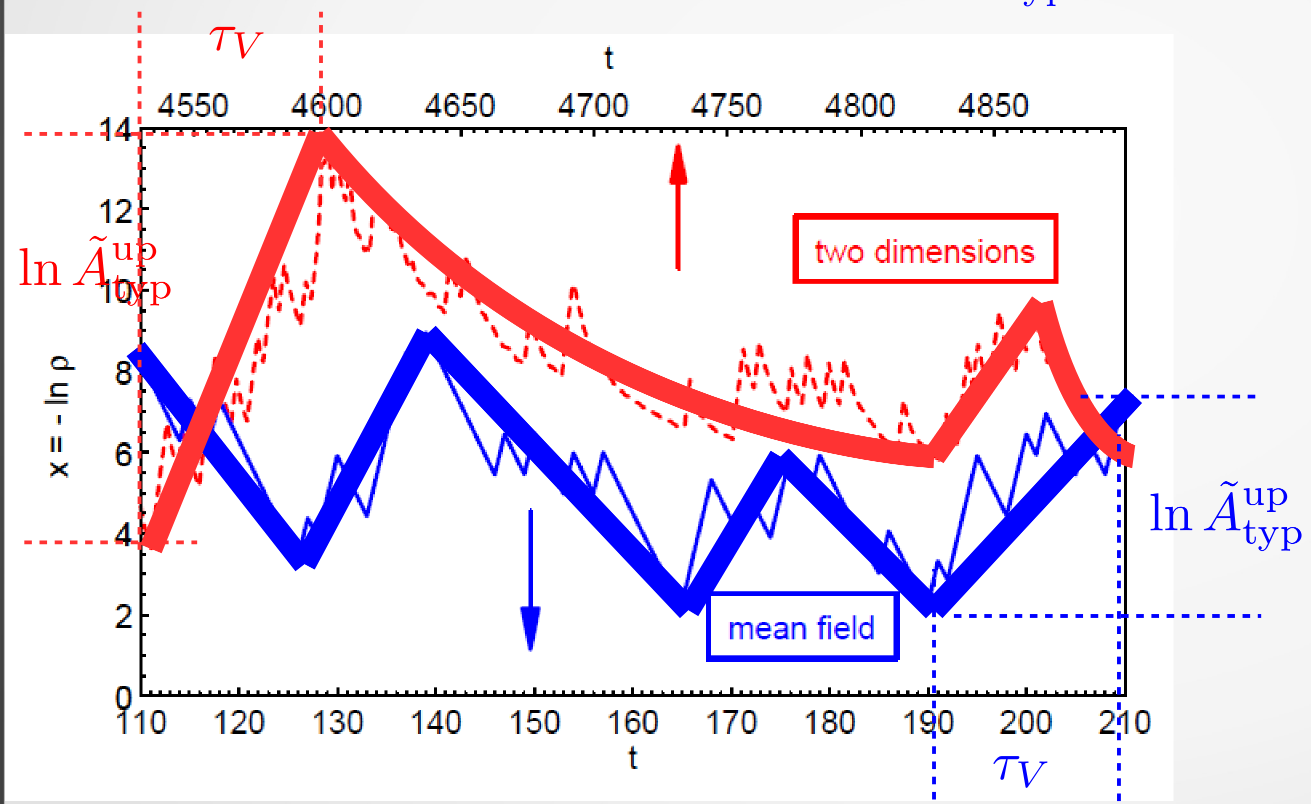
$$\lim_{t \to \infty} \frac{\sigma_\rho}{\langle \rho \rangle} \sim \ln t \to \infty$$

Real-time RG: observables

Keep track of $\tilde{A}^{\mathrm{up/down}}$ and the time intervals Δt

Recall that $\tilde{\rho}_{i+1}^{-1} = \tilde{A}_i \tilde{\rho}_i^{-1} + \tilde{B}_i,$

LIFETIME τ_V = time interval $\Delta\tilde{t}$ when $\tilde{A}^{\mathrm{up}}_{\mathrm{typ}} = V$



Inactive Phase:

$$\tau_V \sim \ln V$$

Active Phase:

$$\tau_V \sim V^{\mathcal{R}_0}$$

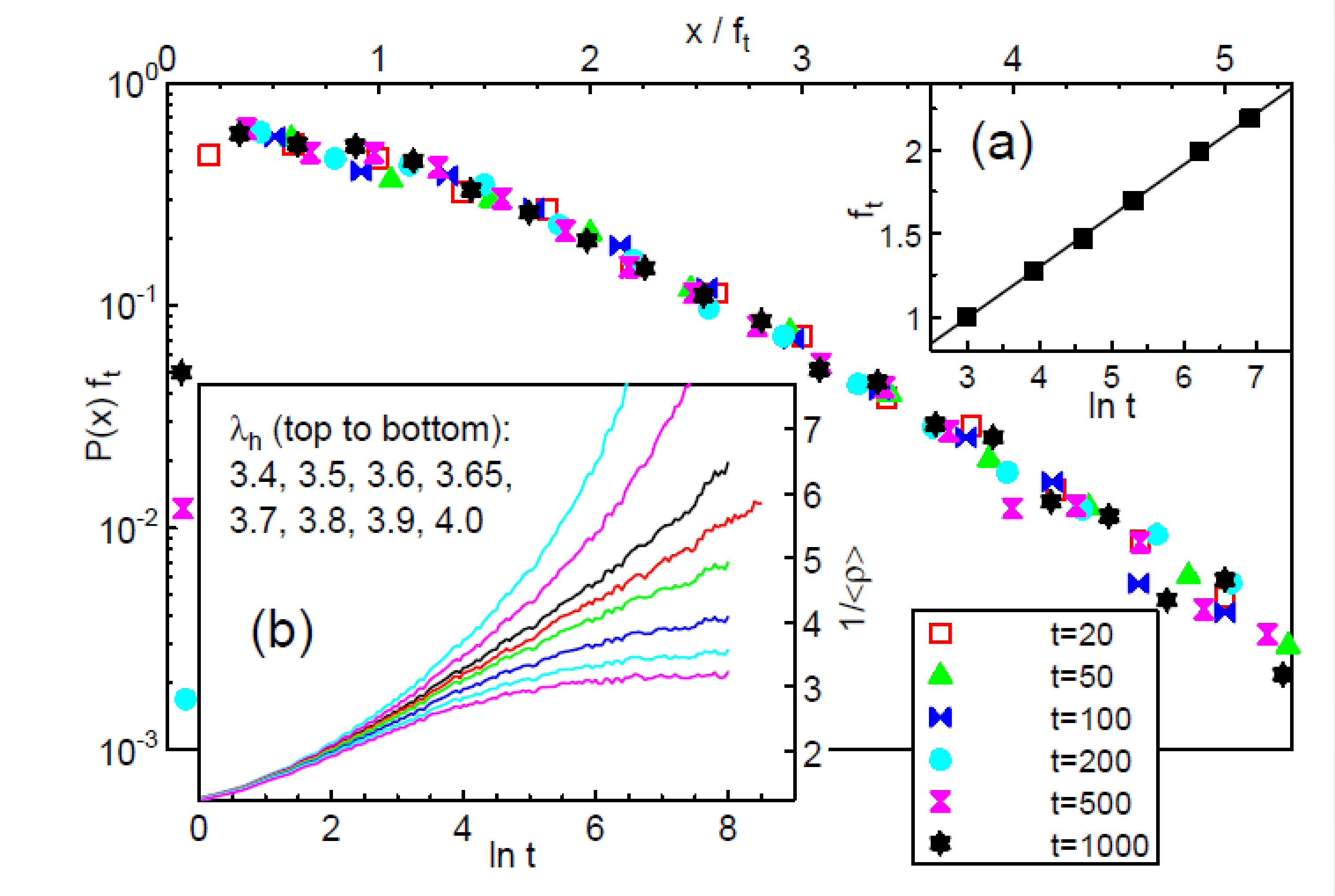
Criticality:

$$\tau_V \sim V^{1/d} = L$$

$$\boxed{z = 1}$$

Real-time RG: comparison with numerics

Criticality
$d=2$



System size
3200×3200

$\mu = 1$

$N = 20\,000$

$W(\lambda) = 0.8\delta(\lambda - \lambda_h) + 0.2\delta(\lambda - \lambda_h/10)$

$\langle \rho \rangle \sim (\ln t)^{-\bar{\delta}}$, with $\boxed{\bar{\delta} = 1}$

In agreement with our prediction

Conclusions

Infinite-noise criticality:

Environmental noise (time fluctuations) on the parameters of the Contact Process yields to a non-equilibrium PT characterized by strong density fluctuations with respect to the many different histories.

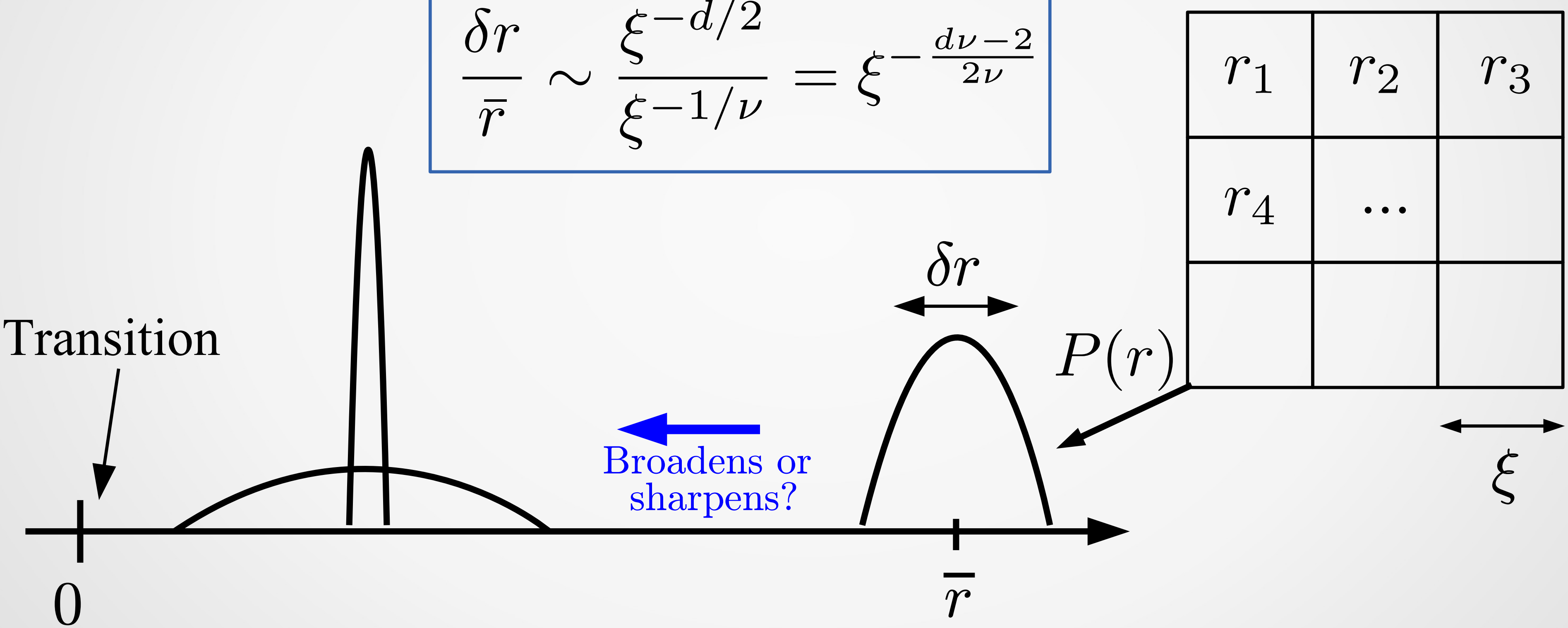
EPL **112**, 30002 (2015)

arXiv:1507.05677

Relevance of disorder: Harris criterion

Harris criterion: $d\nu \geq 2$ - A. B. Harris, J. Phys. C **7**, 1671 (1974)

$$\boxed{\frac{\delta r}{\bar{r}} \sim \frac{\xi^{-d/2}}{\xi^{-1/\nu}} = \xi^{-\frac{d\nu-2}{2\nu}}}$$



- Necessary condition for the stability of the clean fixed point
- Based on the average behavior of disorder

See also Chayes *et al.*, PRL **54**, 2999 (1986)

The ubiquitous DP universality class

Important conjecture: all continuous NEPTs into a single effective absorbing state in the absence of any extra symmetry, long-range processes, or conservation law is in the DP universality class. - Janssen '81 + Grassberger '82

Implications: One can extend the rules of the game

● → ○	Self-destruction
●○ → ●●	Offspring production
●○ → ○●	Diffusion
●○● → ○●○	Coagulation
⋮	

Forbidden process

~~○ → ●~~

CP is the Ising model of the absorbing-state NEPTs

However, although the critical exponents of CP are known numerically (Monte Carlo among other methods), there is no analytical solution for this model

DP universality class: analytical toolbox

DP Langevin equation (master equation)

$$\partial_t \rho(\mathbf{r},t) = (\lambda-\mu)\rho - \lambda\rho^2 + D\nabla^2\rho + \zeta(\mathbf{r},t)$$

Gaussian noise:

$$\langle \zeta(\mathbf{r},t)\rangle = 0$$
$$\langle \zeta(\mathbf{r},t), \zeta(\mathbf{r}',t')\rangle = \Gamma\rho\delta(\mathbf{r}-\mathbf{r}')\delta(t-t')$$

Reggeon field theory (after integrating out ζ)

$$S = \int \mathrm{d}\mathbf{r}\mathrm{d}t\, \bar{\phi}(\partial_t - D\nabla^2 - (\lambda-\mu) + \frac{1}{2}\Gamma(\phi-\bar{\phi}))\phi$$

$\phi(\mathbf{r},t) \sim \rho(\mathbf{r},t)$

$\bar{\phi}(\mathbf{r},t) \equiv$ Martin-Siggia-Rose response field

From here, Green's function and Feynman diagrams

Hamiltonian formulation (H is a generator of a Markov process)

$$\partial_t P = -HP$$

$P \equiv P(s_1,\ldots,s_N,t)$ is the prob. of finding the system in state $(s_1,\ldots,s_N)$ at time t

Steady state – ground state of H

Long-time relaxation – low-energy spectrum of H

For the CP: $H = \mu\sum_i M_i + \sum_{\langle i,j\rangle} n_i Q_j + Q_i n_j$

Price to pay: non-hermitean matrices

 $= |+\rangle$

 $= |-\rangle$

$$M = \begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix}$$

$$n = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad Q = \begin{pmatrix} 1 & 0 \\ -1 & 0 \end{pmatrix}$$

DP and turbulent liquid crystals

Takeuchi *et al.*, PRE **80**, 051116 (2009)

Experimental set up

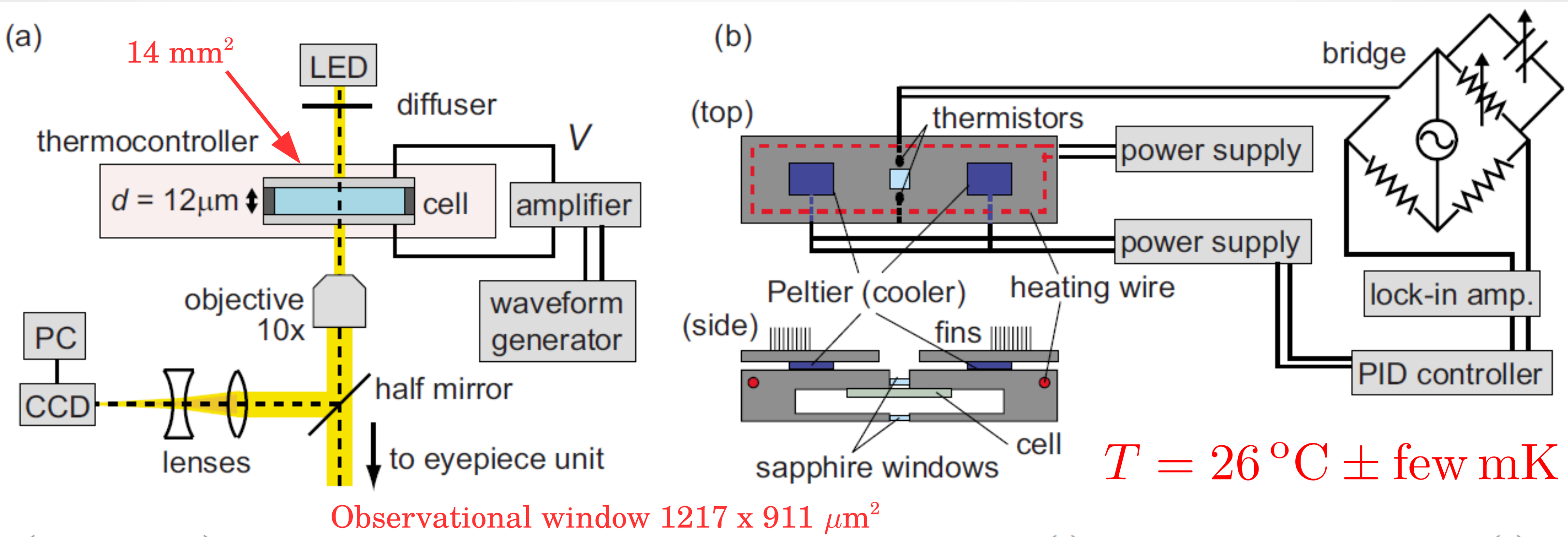


FIG. 2. (Color online) Schematic diagram of the experimental setup in its entirety (a) and for the thermocontroller (b). LED: light-emitting diode, CCD: charge-coupled device camera, PC: computer, PID: proportional-integral derivative. See text for details.

Description:

- Nematic liquid crystal between two transparent electrodes.
- Carr-Helfrich instability for sufficiently high V (and low ω) $\Rightarrow$ a turbulent flow.
- For higher V, new turbulent regimes exhibiting spatiotemporally intermittent dynamics.
- Two topologically distinct turbulent regimes: DSM1 and DSM2 (with disclination deffects).
- DSM2 can disappear into DSM1 and induce the creation of new DSM2 (like a 2+1-d CP).

DP and turbulent liquid crystals

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Acquiring the data

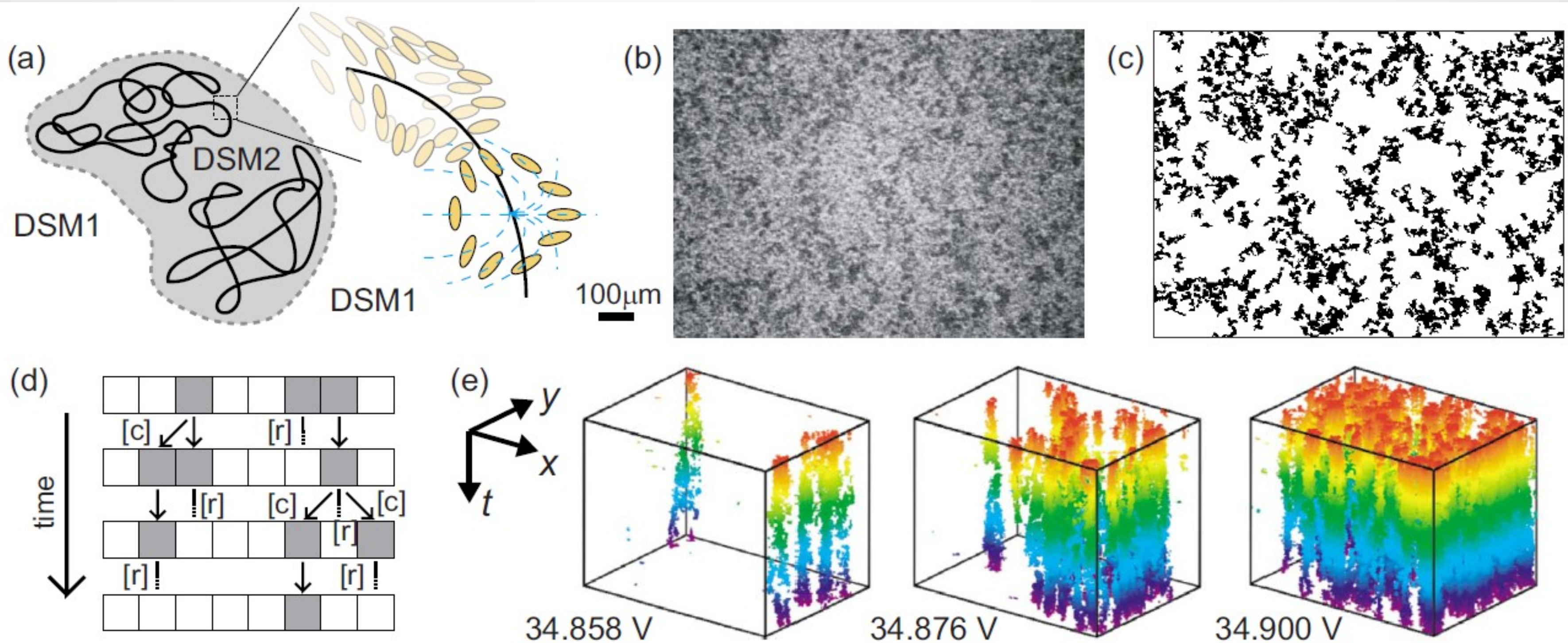

FIG. 1. (Color online) Spatiotemporal intermittency between DSM1 and DSM2. (a) Sketch of a DSM2 domain with many entangled disclinations, i.e., loops of singularities in orientations of liquid crystal. Blue dashed curves in the closeup indicate contour lines of equal alignment. (b) Snapshot taken at 35.153 V. Active (DSM2) patches appear darker than the absorbing DSM1 background. See also movie S1 in Ref. [33]. (c) Binarized image of (b). See also movie S2 in Ref. [33]. (d) Sketch of the dynamics: DSM2 domains (gray) stochastically contaminate [c] neighboring DSM1 regions (white) and/or relax [r] into the DSM1 state but do not nucleate spontaneously within DSM1 regions (DSM1 is absorbing). (e) Spatiotemporal binarized diagrams showing DSM2 regions for three voltages near the critical point. The diagrams are shown in the range of $1206 \times 899\ \mu m^2$ (the whole observation area) in space and 6.6 s in time.

DP and turbulent liquid crystals

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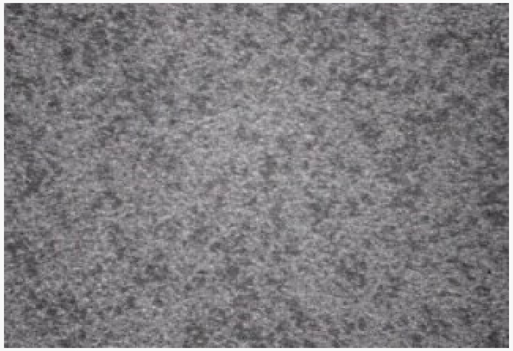
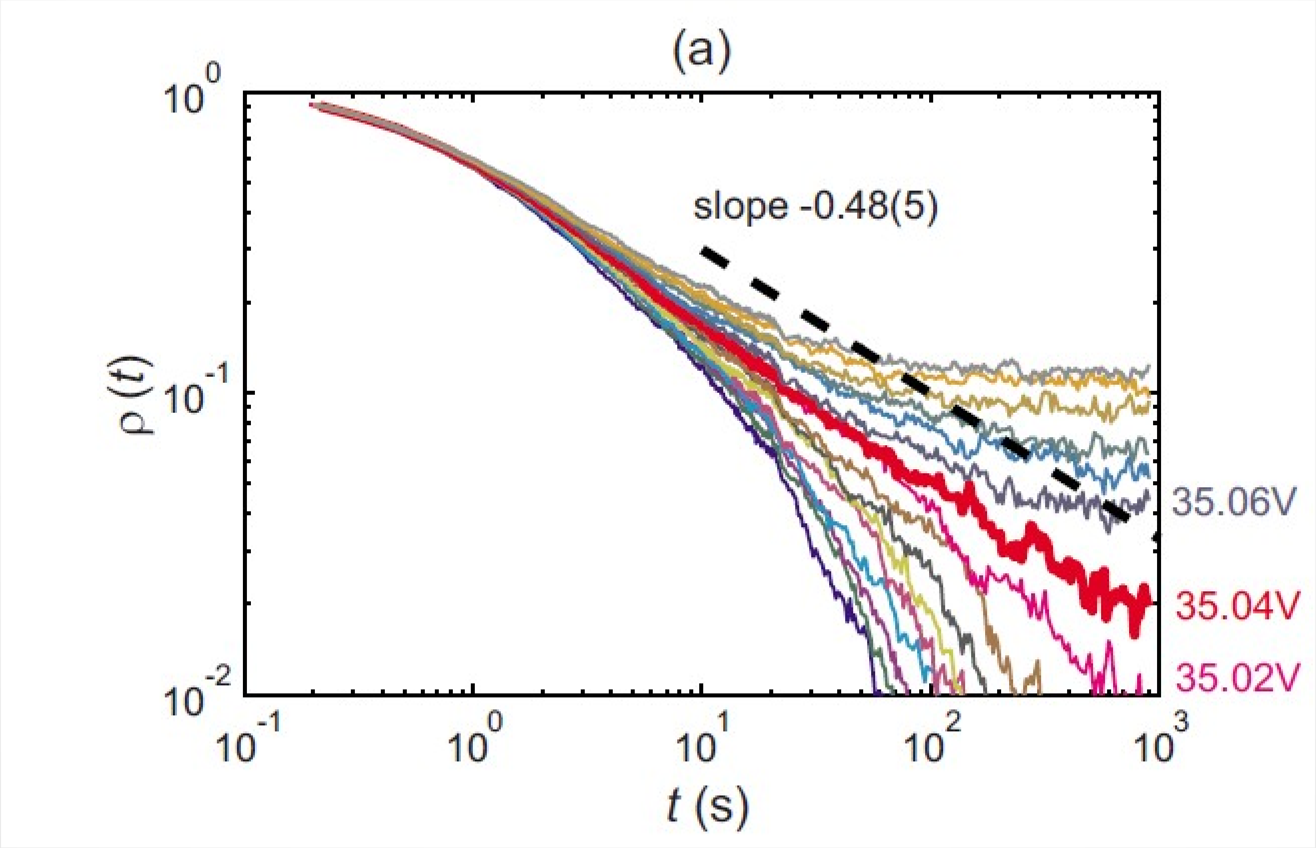
Movie of the turbulent flow: DSM1 (DSM2) regions correspond to lighter (darker) patches.



DP and turbulent liquid crystals

Takeuchi *et al.*, PRE **80**, 051116 (2009)

They confirm 12 critical exponents, 5 scaling functions, and 8 scaling relations



DP and turbulent liquid crystals

Takeuchi *et al.*, PRE **80**, 051116 (2009)

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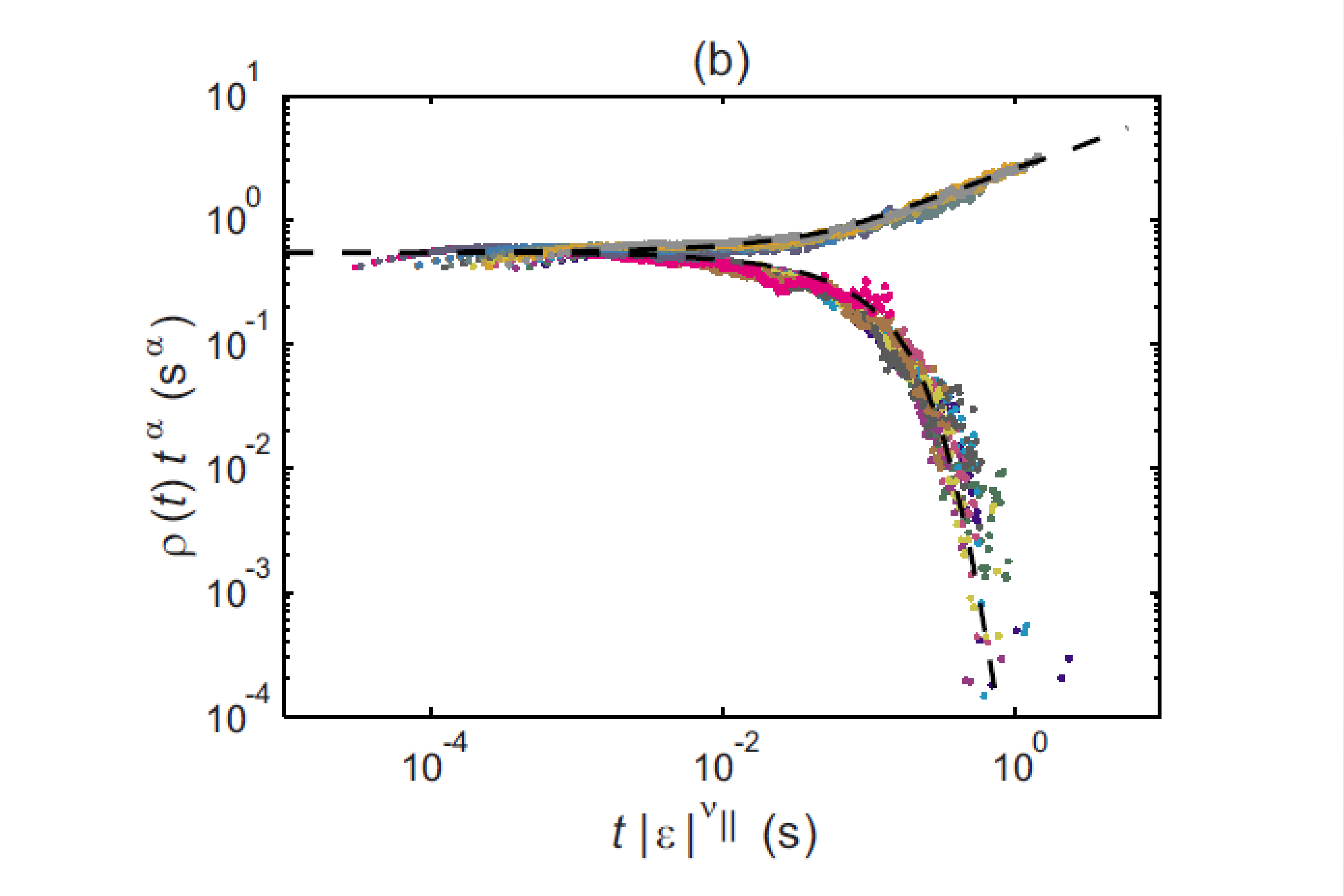

DP and turbulent liquid crystals

Takeuchi *et al.*, PRE **80**, 051116 (2009)

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Exponent		DSM1-DSM2		DP[a]
Density order parameter	β	0.59(4)		0.583(3)
Correlation length[b]	$\nu_\perp$	0.75(6)	0.78(9)	0.733(3)
Correlation time	$\nu_\parallel$	1.29(11)		1.295(6)
Inactive interval in space[b]	$\mu_\perp$	1.08(18)	1.19(12)	1.204(2)[c]
Inactive interval in time	$\mu_\parallel$	1.60(5)		1.5495(10)[c]
Density decay	α	0.48(5)		0.4505(10)
Local persistence	θ_l	1.55(7)		1.611(7)[d]
Aging in autocorrelator	b	0.9(1)		0.901(2)
	λ_C/z	2.5(3)		2.583(14)
Survival probability	δ	0.46(5)		0.4505(10)
Cluster volume	θ	0.22(5)		0.2295(10)
Cluster mean square radius	ζ	1.15(9)		1.1325(10)

The ubiquitous DP universality class?

Examples:

- Epidemics
- Cellular automaton
- Catalytic chemical reactions
- Spatio-temporal intermittency in turbulent fluids
- Growing interfaces, etc. - H. Hinrichsen, Adv.Phys. **49**, 815 (2000)
- Driven polymeric suspensions
- Dynamics of SC vortices
- Mutualism in bacterial colonies

Although the DP universality class seems to be ubiquitous…

"… *there is still no experiment where the critical behavior of DP was seen. This is a very strange situation in view of the vast and successive theoretical efforts made to understand it. Designing and performing such an experiment has thus top priority in my list of open problems*".

- P. Grassberger, Directed percolation: results and open problems, in Nonlinearities in complex Systems, edited by S. Puri *et al.* (1997).
- See also H. Hinrichsen, Braz. J. Phys. **30**, 69 (2000)

The ubiquitous DP universality class?

The culprit: DISORDER

Harris criterion: spatial quenched disorder is relevant if $d\nu_\perp \leq 2$

temporal disorder is relevant if $\nu_\parallel \leq 2$

spatial quenched disorder: rates are time-independent random variables.

temporal disorder: rates are space-independent random variables.

critical exponent	MF	IMF [155]	$d=1$ [168]	$d=2$ [125]	$d=3$ [170]	$d=4-\epsilon$ [171]
β	1	1/2	0.276486(8)	0.584(4)	0.81(1)	$1-\epsilon/6-0.01128\,\epsilon^2$
$\nu_\perp$	1/2	1	1.096854(4)	0.734(4)	0.581(5)	$1/2+\epsilon/16+0.02110\,\epsilon^2$
$\nu_\parallel$	1	3/2	1.733847(6)	1.295(6)	1.105(5)	$1+\epsilon/12+0.02238\,\epsilon^2$
z	2	3/2	1.580745(10)	1.76(3)	1.90(1)	$2-\epsilon/12-0.02921\,\epsilon^2$
δ	1	1/2	0.159464(6)	0.451	0.73	$1-\epsilon/4-0.01283\,\epsilon^2$
θ	0	1/2	0.313686(8)	0.230	0.12	$\epsilon/12+0.03751\,\epsilon^2$
γ	1	3/2	2.277730(5)	1.60	1.25	$1+\epsilon/6+0.06683\,\epsilon^2$
υ	1	1/2	0.82037(1)	0.88	0.94	$1-\epsilon/12+0.03317\,\epsilon^2$
σ	2	2	2.554216(13)	2.18	2.04	$2+\epsilon^2/18$

Spatial quenched disorder is relevant in most of the interesting cases $(d=1,2,3)$

Temporal disorder is relevant in all cases